Aerobatic Capabilities of “Marginally Aerobatic” Airplanes

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Note 1: This is an informal paper that I wrote in my spare time for fun. It does not represent the views of my employer, my flight instructors, the FAA, or anyone else.
It has not been reviewed by peers or experts. There may quite possibly be errors in my math, bugs in my code, and/or poor assumptions in my physics.
If you would like to provide any feedback, please do feel free to email me.

Note 2: Do not disobey the limitations set by the manufacturer of any aircraft! Respect the POH and the placards on the panel, including the ones that prohibit aerobatic flight on most airplanes.

Open up any book on airplane performance, and various equations are derived which allow for the calculation of the climb rate, climb angle, required runway length, range, and other specifications for a given airplane (with a given weight and at a given density altitude). Airplane design books give similar equations that make it possible to determine (given some parameters such as airplane weight) how much wing area would be needed in order for takeoff and landing to be possible at a certain runway, how much fuel would be needed to carry a certain payload a certain distance, and how much engine power would be needed in order to achieve a certain climb rate.

But what about aerobatics? What can a given airplane do? Not just “What is allowed by the POH and by the placards, or lack thereof?”, but rather: What would be physically possible? And what characteristics – such as stall speed, structural strength, roll rate, and maximum possible speed – are needed for certain maneuvers to be “at least barely possible”? How would one calculate the “margins” regarding how close a given maneuver would push a given airplane towards one of the edges of its V-G envelope?

There are few papers in the aeronautical engineering literature that concern themselves with determining whether a given airplane is capable of aerobatics. Either an airplane is clearly capable of aerobatics (such as a fighter or a Pitts), or it is “marginal” (such as an airliner or an LSA), and “marginal” airplanes are not certified for aerobatics so the question is never investigated by professionals in the field.

However, laypeople often ask: Would an airliner be capable of a roll, a loop, of sustained inverted flight? The famous Text Johnston 707 roll in 1956, and various action movie sequences, keep such questions in the public consciousness.

I would like to address these questions, not just for jetliners but also for piston-powered trainers, for twin-propeller airplanes, and generally for any airplane.

From a test-pilot’s point of view: What parameters would allow you to calculate, analytically, just how safe/dangerous it is to take a given airplane and perform a loop, a roll, inverted flight, etc.?

What follows are some attempts at quantifying aerobatic capabilities, not so as to maximize them in tactical or aerobatic-competition aircraft, but primarily so as to evaluate what “marginal” airplanes (such as airliners and piston-powered trainers) can do, and what the relationships are between airplane characteristics (stall speed, VNE, limit load, roll rate) and aerobatic capabilities.

Aileron rolls

As we all know, there are various kinds of rolls, from barrel rolls – whose “centrifugal” effect presses the pilot against the seat the whole time – to slow rolls – whose sustained inverted segment has the pilot hanging from the straps. The easiest roll is exactly in-between: the aileron roll, which happens during approximately zero-g flight.

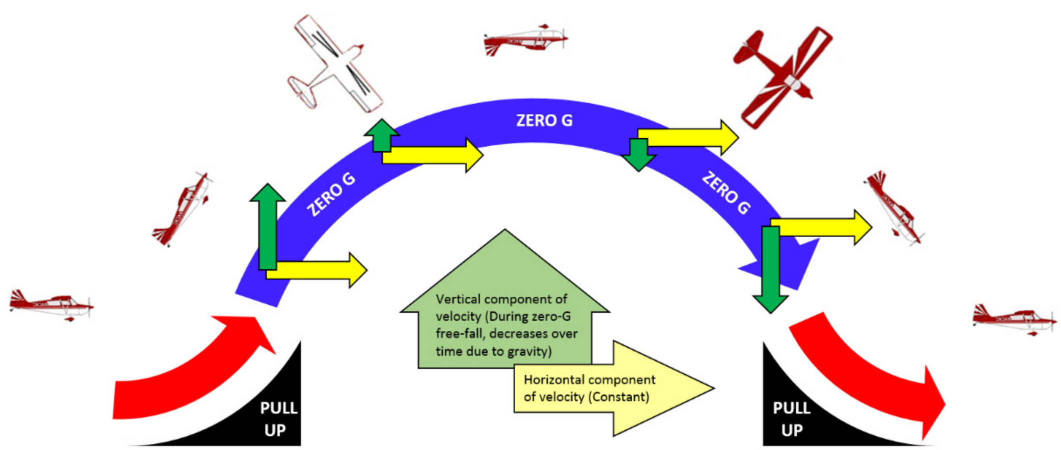
Like a “Vomit Comet” parabola, the aileron roll starts with a pull-up into climbing fight, then a ballistic segment during which the roll is performed until the nose is pointing below the horizon, followed by a pull-up returning to horizontal flight.

Which airplanes are capable of aileron rolls?

This question can be broken down into: For how long can a given airplane fly a zero-g ballistic parabola? And: Is that long enough, given the airplane’s roll rate, to perform a complete 360-degree roll?

1) <u>Zero-g time</u>

The time that can be spent in a zero-g ballistic trajectory depends, of course, on the vertical component of the velocity at the end of the initial pull-up. Once at zero-g, gravity will decrease the vertical component of the airplane’s velocity by 9.81 meters per second, per second. This will happen until the vertical component of velocity reaches zero, at which point the airplane will reach the top of the parabola… and then it will continue for roughly the same amount of time, until the airplane has a downwards angle and velocity that is roughly equal and opposite to the upwards angle and velocity at the end of the pull-up / start of zero-g. (This is your basic “projectile motion” problem from high-school physics).



What is the velocity of the airplane at the end of the pull-up and at the start of the zero-g phase?

A very simple initial model may be to imagine the airplane instantaneously converting all of its forwards velocity into upwards velocity, going up (or at least diagonally) at the same speed (i.e. velocity magnitude) as it had when traveling forwards. This would indeed maximize time at zero-g.

But is it realistic?

No, it is not realistic, for two reasons: (1) Due to critical limits in angle of attack (stall), structural strength (wings braking off), and the human pilot (loss of consciousness), the airplane can only pull so-many Gs. This means the pull-up will have some non-zero radius, causing a gain in altitude, causing some kinetic energy to be converted to potential energy. Also, (2) Due to induced drag, the extra lift during the pull-up will increase the drag of the airplane, causing it to lose speed. Therefore, the speed at the end of the pull-up will be lower than the speed at the start.

[Specifically, one of the equations describing steady circular motion is $A=V\Omega$. Plugging in some numbers will reveal that, for every 100mph of speed V, and for every G of acceleration A in addition to the 1g needed for level flight, pitching up takes about 12° per second. In other words, the pitch-up points the nose upwards at roughly 12° per second, per extra G of acceleration, for each 100mph. For example, if traveling at 500mph and performing a 1.8g pitch-up to 30° (typical for the Vomit Comet), that pitch-up will take about 15 seconds. This is because 1.8g-1g=0.8g, and 0.8 times 12°/s gives about 10°/s, and 30° divided by 10°/s is three seconds, but this is for every 100mph so at 500mph the 3 seconds become (3*500/100)=15 seconds].

In addition, the angle of the pull-up rarely exceeds 30° to 45°, in practice. Any steeper, and the airplane would become severely slow at the top (as we will quantify when we model loops). This would cause a reduction in roll rate (due to decreased aerodynamic forces on the ailerons), and it would require the airplane to change from “pointing upwards” to “pointing downwards” relatively abruptly near the top.

Long story short, only about half of the airplane’s entry speed becomes vertical velocity.

9.8m/s is about 22 mph, which is the vertical speed taken away by gravity every second. If horizontal velocity could be instantly turned into vertical velocity, then for every 110mph of speed, the airplane would get five seconds going up, and five seconds coming down, i.e. ten seconds of free-fall. However, due to the “half” factor we just discussed, as well as speed lost during the pull-up, in practice an airplane will only get 4 or 5 seconds of zero-g time for every 100mph of entry speed.

2) <u>Rolling</u>

The question then becomes: Is that enough to roll 360°?

The answer is nearly always: Yes.

An RV-6 going at 150mph will have about 7 seconds to roll all the way around. That is plenty of time.

A Cessna, LSA, Cub, or Citabria going at 100mph will have 4 or 5 seconds to roll 360°. Will that be enough? Just barely. Typically, when performing aerobatics in under-powered single-engine trainers, the pilot starts out by getting into a slight dive and picking up some extra speed at the start, because these airplanes’ VNE (never-exceed speed, typically 15% below the speed at which flutter might occur) is significantly higher than speeds that can achieved in level flight. Another option is to point the airplane up 45° or more. This risks an inverted spin at the top of the maneuver when the speed is slow, and/or exceeding VNE during the steep descent near the end.

An airliner going at 500mph will have about 20 seconds of zero-g (as shown by the Vomit Comet) to roll all the way around. As Tex Johnston showed us in 1956; That is plenty of time to roll a 707. Rebecca Wallick, when interviewing Boeing’s test pilots for her book (including her father Lew), discovered that, in court, none of them were willing to deny having rolled “their” prototype airliners (from the 727 to the 767). Plenty of pilots have rolled B-1 bombers during the airshows at Edwards AFB over the years, and a roll is a standard part of the C-27 airshow demo. Skilled airshow pilots have rolled old Beech 18s and new single-engine Cessna TTXs. Even the Vulcan bomber was rolled once prior to its retirement, and at least one video exists of a V-22 being rolled.

In short, ***all but the most sluggish*** (low roll rate) ***airplanes can perform aileron rolls***. When pulled up into a ballistic parabola, an airplane can easily get about ***5 seconds of zero-g time for every 100mph of initial horizontal speed***, and that time is enough for nearly all airplanes to roll 360°.

Loops

The main complication that comes from performing loops is the fact that they are very tall maneuvers. While going up into a loop, the airplane primarily relies on inertia, with engine thrust barely helping (due to being largely canceled out by drag). This means that the speed at the top of the loop becomes exceptionally slow; Most of the airplane's kinetic energy is converted to gravitational potential energy on the way up. The ability of an airplane to go all the way around a loop without "falling off the top" relies on the airplane's ability to pull Gs and, most importantly, on the maximum speed that can be reached horizontally prior to the start of the loop. The faster an airplane can go at the bottom, the more speed is "left over" at the top. You may have seen stunts where motorcycles or skateboarders pick up speed and then go all the way around a loop-shaped ramp; Airplane loops are governed by very similar principles.

Because this part of the paper will be quite long, it would be appropriate to start by summarizing the final results: The more accurately we model the loop (e.g. teardrop-shaped rather than circular), the easier the G requirements become. In the end, the required centripetal acceleration (pull-up force) is only about 2g, which most airplanes are easily capable of. It is the speed performance, however, that will emerge as the key limitation. An airplane that can pull 3g must enter the bottom of the loop with an airspeed that is at the very least about 1.7 times its stall speed. An airplane that can only pull only 2 to 2.5g must enter the bottom of the loop while flying at twice its stall speed, at the very minimum. Any slower, and the airplane's speed will drop to zero around the time the nose is pointed straight up, if not earlier. The only way to make it all the way around is to start with plenty of speed.

Especially if starting in a slight dive, most small propeller airplanes can easily and safely fly at 2 or 3 (sometimes 4 or 5) times their stall speed, and can pull 3 to 4g (even more if not fully loaded). This means that Cessnas and LSAs are easily capable of performing loops. However, large transport jets such as airliners do struggle to fly at just twice their [flaps-up] stall speed, and risk damaging the structure if more than 2.5g are pulled. So, as we will see, their looping capabilities are extremely marginal. They can only perform a loop if flown very precisely, along the very top of its V-G envelope: The pilot must pull just under 2.5g at high speed (bottom of the loop), then keep the angle of attack just below the stall at slow speeds (upper part of the loop), or the airplane will run out of speed before reaching the top.

Let's do the math. There are a few increasingly accurate ways to model loops:

1) Circular loops, with no loss or gain in energy
2) Constant-G teardrop loops, with no loss or gain in energy
3) "Constant G above VA, critical alpha below VA" teardrop loops, with no loss or gain in energy
4) "Constant G above VA, critical alpha below VA" teardrop loops, with changes in speed causing changes in excess thrust, thus also causing energy to be lost or gained.

We will mathematically model the first three, and leave the fourth as an exercise to the reader (with many hints).

1) <u>Circular loop</u>

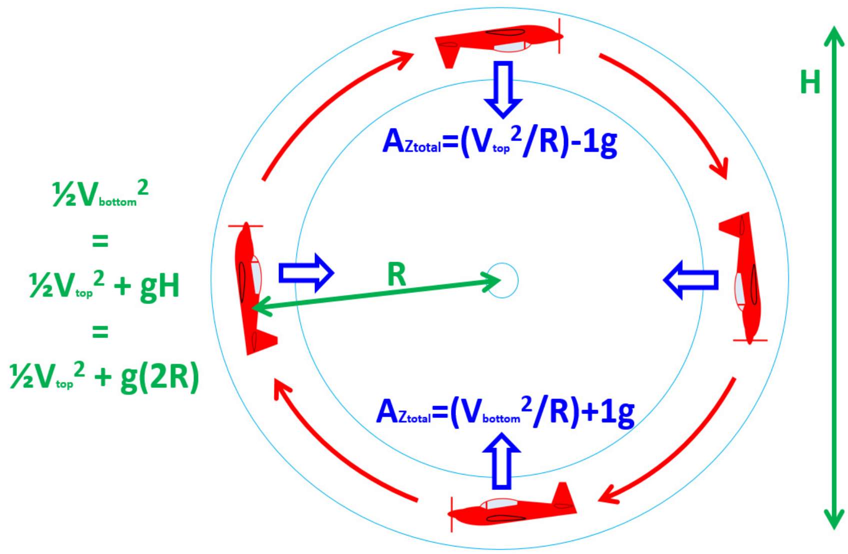
The circular loop is a relatively straightforward high-school physics problem. (However, as we will soon see, it is almost uselessly unrealistic for aeronautical applications). All that it requires is an understanding of kinetic energy, gravitational potential energy, and circular motion.

The question we must answer is: What is the relationship between the entry speed, the loop radius, and the centripetal acceleration? In order to have enough energy at the bottom to make it to the top, what speeds are necessary and how many Gs are experienced while pulling up along the circular trajectory?

The two key equations here are

- $mgH= ½mV^2$, which correlates how much kinetic energy is converted to potential energy (i.e. how much speed is lost) when a body increases its altitude by H, and…

- $A_{centripetal}=V^2/R$, which gives us the centripetal acceleration as a function of speed and radius.



The first equation can be simplified: Divide by ½m, solve for V, and you get $\sqrt{(2gH)}=V$. When discussing the speed at the top of the loop, the altitude gained is two loop radii: H=2R . Drop an object (or allow it to roll down a hill) from an altitude of H to the ground (altitude zero), and it will hit the ground with a speed of $\sqrt{(2gH)}$.

The second equation tells you how many Gs are associated with taking a circular turn of a given radius at a given speed, be it in an airplane or a car or a satellite or any other object not going in a straight line. While pulling up in an airplane, however, this calculation gets a little trickier when you take into account the effect of gravity. The airplane needs 1g just to sustain level flight. At the beginning and end of the loop, the centripetal acceleration (the acceleration pulling the airplane and pilot into the circular loop) is the total acceleration being felt by the airplane and pilot, MINUS the one g for level flight. At the top of the loop, on the other hand, the situation is easier: The centripetal acceleration pulling the airplane and pilot into the circular loop is the total acceleration being felt by the airplane and pilot, PLUS one g. The airplane and pilot may be experiencing almost zero-g at the top of the loop, but the airplane trajectory will still follow that circular arc for a moment, thanks to gravity. And finally, when the airplane is pointed straight up or straight down, gravity pulls only straight back or straight ahead, having no effect on the forces that keep the airplane turning into the circle.

Generalizing, $A_{Ztotal}= A_{centripetal}+1g*\cos\theta$, where θ is the angle along the loop (zero at the bottom, 90° when going straight up, 180° at the top, and 270° when coming straight down). The pilot and airplane experience A_{Ztotal} , and the equation for circular motion ($A=V^2/R$) is determined by $A_{centripetal}$. (Each "g" is an A of $9.81 m/s^2$).

The effect of gravity on centripetal acceleration, and the fact that the airplane slows down as it goes uphill, lead us to ask the following question. Given the airplane's initial speed and the number of Gs it can pull: Will the airplane have enough speed to make it to the top of the circle? Or will it run out of speed before getting to the top?

There are two ways to answer that question, each of which places a different requirement on the centripetal acceleration.

1A) <u>Rigid circular loops</u>

The first way to approach this physics problem is to require that the speed should simply not drop to zero before reaching the top. This is unrealistic for aeronautical applications, but it is simple and therefore a suitable starting point. This model assumes that as long as the airplane (or whatever object) has *some* speed, it will keep moving along a circular path. This describes scenarios such as a vehicle attached to a rail (e.g. the cars of an amusement park ride with a circular loop), a weight at the end of one or more rigid arms (like a circus “Russian swing” or an Estonian kiik swing), or an object moving inside a pipe with a circular loop. One way to characterize this set of situations is by their “worst case scenario”: The object reaches zero speed right at the top of the loop and gets stuck there until it is nudged one way or the other.

(Note: This may seem like an appropriate way to model a “pirate ship” amusement park ride like the one pictured to the right. However, these rides have counterweights. This means that the center of gravity is located just below the axle at the center of the circle, and so the CG only moves up and down a short distance, while the passengers move up and down a large distance. Moving the CG this short distance requires much less kinetic energy, so the ride can be moving quite slowly – and not pulling a whole lot of Gs – at the bottom and still easily make it over the top. We shall disregard this trivial example and only model situations where the CG itself has to go up the entire distance from the bottom of the loop to the top of the loop. No counterweights).

Assuming that the kinetic energy at the bottom is at least the potential energy at the top (i.e. $mgH= ½mV^2$) and that the height H is twice the radius R, then the speed at the bottom is at least $\sqrt{(2g*2R)}=V=\sqrt{(4gR)}$

The centripetal acceleration at the top is at least zero (due to at least zero speed). So the total acceleration at the top, A_{Ztotal} , can be as low as minus 1g (i.e. hanging upside down at the top of the loop).

The centripetal acceleration at the bottom is at least $A=V^2/R=[\sqrt{(4gR)}]^2/R = 4g$ (required to turn into the circle). The total acceleration at the bottom is that plus $1g*\cos(0) = 5g = A_{Ztotal}$.

It’s interesting how, no matter how tall a circular-loop pipe or a Russian swing or a circular roller-coaster loop is: If the speed at the top is zero (e.g. if something is carefully balanced at the top of the loop and then is gently pushed down one direction), the total acceleration at the bottom will be 5g. A taller loop will involve a higher speed at the bottom but also a wider circle with less-intense centripetal acceleration “per speed”, and those will balance out exactly, always giving a 5g peak total acceleration at the very bottom.

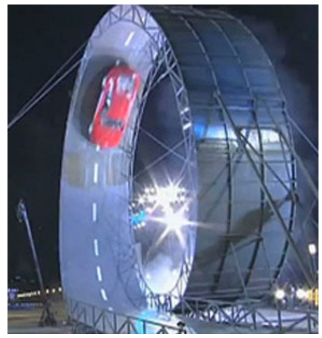
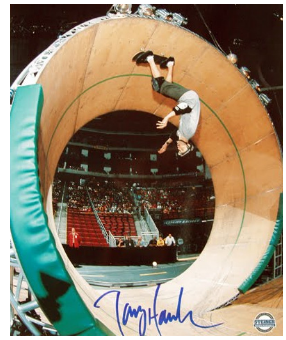
And because this is reversible (assuming that the friction is either zero or is canceled out by some kind of engine): Entering a circular loop at the correct combination of speed and loop radius to generate an initially-5g acceleration will result in the speed reaching zero at the very top.

This, of course, is the “bare minimum”: ***Pulling any more than 5g at the start of a rigid circular loop (by going faster and/or turning into a tighter circle) will mean that some speed is still “left over” at the top***.

1B) Non-rigid circular loops

The preceding discussion on rigid loops may be useful for circus acrobats on their Russian swings, and for modeling many amusement park rides. However, no airplane can fly at zero airspeed at the top of the loop, and very few airplanes can lose speed going up a circular loop and still pull off negative-one-g flight at the top. Most loops – be it airplanes, swings, or a loop-shaped track with a stunt motorcycle or skateboarder or toy car in it – must be performed without the total acceleration dropping below zero, i.e. with no less than 1g of centripetal acceleration at the very top (so that the vehicle is “weightless” for just a moment at most at the very top). Otherwise, the vehicle “falls off” the top of the loop.



So instead of the speed at the top needing to be at least zero, for a non-rigid loop, the speed at the top needs to be at least the speed that requires 1g of centripetal acceleration to pull the vehicle into the loop’s circular arc.

This means that at the top , $A_{centripetal} = V^2/R \geq 1g$, so V_{top} needs to be at least $\sqrt{(gR)}$.

At the bottom , $½mV_{botom}^2 = ½mV_{top}^2 + mgH$ i.e. $½V_{botom}^2 = ½[\sqrt{(gR)}]^2 + g(2R)$

$V_{botom} = \sqrt{\{ [\sqrt{(gR)}]^2 + 2g(2R) \}} = \sqrt{\{ gR + 4gR \}} = \sqrt{\{5gR\}}$ (Recall that for the rigid-arm loop, it was $\sqrt{\{4gR\}}$).

As for the acceleration: $A_{Ztotal@bottom} = V_{botom}^2/R + 1g = 5g + 1g = 6g$.

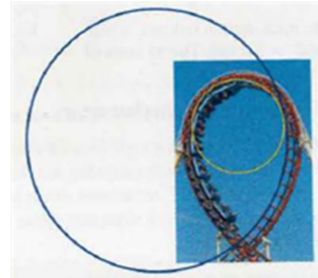
This means that, ***in order to fly (or drive, or swing) a truly circular loop without the Gs going negative at the top, you need to pull at least 6g at the bottom.***

(Similarly, in order to get a chain swing or a stunt skateboarder or motorcycle or car to do a 360° loop, it must be going fast enough at the bottom that the combination of radius [e.g. the chain length] and speed should make for $A_{Ztotal} = V^2/R + 1g \geq 6g$).

This model, although more useful, neglects two factors, causing it to be very conservative (i.e. In reality, airplanes need much less than 6g to do a loop) The first factor: As the airplane slows down on its way up the loop, the speed will become less than the terminal speed at that engine setting, which means the airplane will have “excess thrust” for that speed, and so it will not lose speed quite so fast. This is one reason why these 5g and 6g numbers are conservative: In a powered airplane, the speed (and thus the centripetal acceleration) at the top will be slightly higher than these numbers (thanks to engine thrust on the way up), so it’s not necessary to pull 6g at the bottom.

The other factor being neglected is the fact that…

2) <u>Constant-G teardrop loops</u>



… as speed decreases on the way up the loop, the radius of turn can decrease without the Gs increasing. In a circular loop, you have to pull many Gs at the very bottom but then the Gs die down until they are zero at the top. But by keeping the Gs high by tightening the radius as you go up the loop, the loop becomes much less tall, requiring much less energy (and thus less speed and Gs) to get all the way around. The loop will then no longer be a circle. Rather, it will be a teardrop shape. Notice how nearly every roller-coaster loop has a teardrop shape, and most loops flown by aerobatic airplanes at airshows have a teardrop shape. Near the top, slower speeds are combined with tighter turn radii in order to keep the Gs roughly constant all the way around. Even at the top of the loop, the pilot is pressed firmly "down" into the seat.

As we saw, $A_{Ztotal} = A_{centripetal} + 1g*\cos\theta$ where $A_{centripetal} = V^2/R$

So now say that, instead of R being constant and A_{Ztotal} changing as you go around the loop (from 5g or 6g at the bottom, to 0g or -1g at the top), let's hold A_{Ztotal} constant (i.e. pulling up as hard as you can, the whole way around) and ask: How does the radius change as you go around the loop?

Solve $A_{Ztotal} = V^2/R + 1g*\cos\theta$ for R and you get $R = [\ A - g \cos\theta\ ] / [\ V^2\ ]$

The speed changes as a function of height, just like before, as $\frac{1}{2}mV^2$ changes into mgH. So the decrease in V^2 matches the increase in 2gH. So the square of the speed at any height is $V_{bottom}^2 - 2gH$. In other words,

$$R = [\ A - g \cos\theta\ ] / [V_{bottom}^2 - 2gH\ ]$$

This is a function of the nose-up angle θ (zero at the bottom, 90° when going straight up, 180° at the top, etc.) and of the height H at each point along the loop.

But the change in angle per time, Ω, is A/V i.e. $A/[V_{bottom}^2 - 2gH\ ]$…

And the change in H per time, the vertical component of V, is $[V_{bottom}^2 - 2gH\ ]\sin\theta$ …

So if you try to set up these equations and solve them analytically, you get some really ugly differential equations that require the use of vector calculus and differential geometry. It is doable, if you really like math. Look up clothoids, cornu spirals, Euler spirals, and so on:

http://physics.gu.se/LISEBERG/eng/loop_pe.html http://datagenetics.com/blog/march42014/index.html

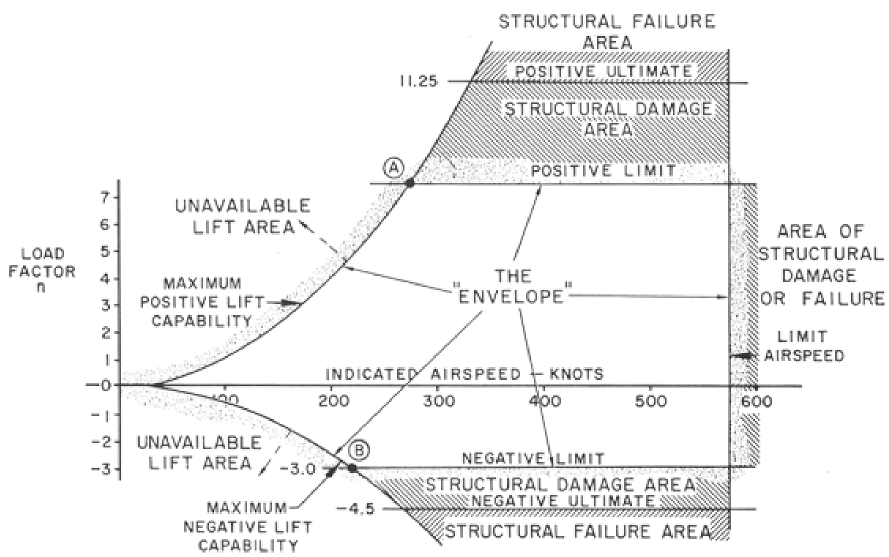
Personally, instead of actually doing the math (and in order to keep the physics and math here at a high-school level), I would prefer to solve these equations using numerical analysis, i.e. an explicit iterative method using single linear steps, maybe with multiple stages to increase accuracy (which is the nice way of saying "Brute force computation instead of exact equations"). And I will, in a minute. But before I do, I would like to acknowledge one more imperfection of this mathematical model.

3) <u>"Constant G above VA, critical alpha below VA" teardrop loops</u>

The final tweak we will make to our loop math is the fact that it is not always possible to pull as many Gs as you want. The airplane slows down on its way up the loop. Once it slows down below the maneuver speed, VA, pulling "limit load" (the maximum number of Gs that the wing structure can safely take) would cause the airplane to stall. So instead of pretending that the airplane can pull maximum Gs all the way around the loop, we should find the height at which the speed will drop below VA, and when below that speed (i.e. when above that height) we should only pull as many Gs as possible *without stalling*.

Let's pretend that, once we reach VA, we want to maintain a certain angle of attack (such as, for example, the angle that gives CLmax for your wing's airfoil at applicable Reynolds numbers). Let's say that, at the stall speed, flying at that angle of attack provides 1g worth of lift. As you fly faster and hold that angle of attack, more lift is generated because the dynamic pressure goes up, and the dynamic pressure is proportional to the square of the speed: Fly at 1.1 times your stall speed, and you can pull 1.21g before you stall. Fly at 1.2 times your stall speed, and you can pull 1.44g before you stall. Fly at 3 times your stall speed, and you can pull 9g before you stall (wing structure permitting).

So, at speeds below VA, the maximum number of Gs that can be pulled is 1g times the square of the ratio between the speed and the stall speed. (VA is the speed at which this quantity, the maximum number of Gs that are aerodynamically possible, exceeds the structural limit load).

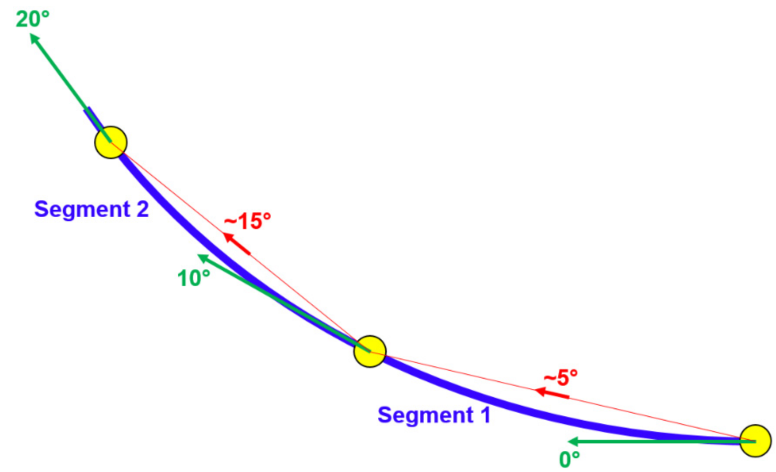


Ok, so let's do a numerical analysis.

Our airplane can be safely flown at some maximum speed (say, VNE, which is typically set at about 85% of the flutter speed), and can safely pull some number of Gs (say, the limit load, which is either the load where the structure starts yielding, or 2/3 of the load where the structure experiences catastrophic failure, whichever is lower), and has some stall speed (at which the wings can only generate 1g worth of lift when at the angle of attack that generates the most lift). Those are the inputs. The question is: Can the airplane do a loop?

Let's model the airplane's loop as a series of linear (straight-line diagonal) increments. We'll split it into 10° increments of θ, so we will need about 36 of those increments.

The first increment will start with the airplane going horizontally and pulling to 10°. Our linear model will treat this segment as a small 5-degree slope. The next increment starts when the airplane is at 10° nose-up and goes to 20°, i.e. we will treat it as going uphill by a 15° slope. The following increment after that will start when the airplane is at 20° nose-up and goes to 30°. And so on, 360° around.



The speed at the start of each increment is the speed at the end of the previous increment (which we will calculate in a minute).

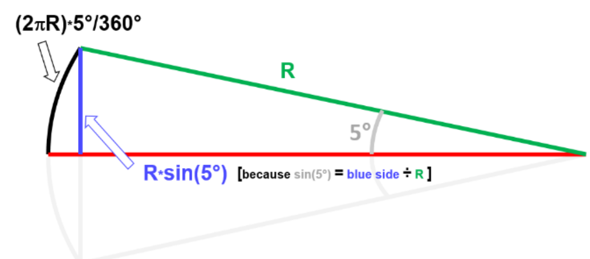
If above VA, the total Gs being pulled are the limit load.

If below VA, the total Gs being pulled are the max aerodynamically possible Gs, which is the square of the ratio between the current speed and the stall speed.

The centripetal acceleration are the total Gs being pulled, minus one times the cosine of the angle. (For example: While flying horizontally, the centripetal A is the total A being experienced by the airplane and pilot, minus the 1g needed for level flight. At the top of the loop, the centripetal A is the total A being experienced by the airplane and pilot, plus the 1g from gravity helping the airplane follow the arc of the loop even if the pilot feels weightless. And so on).

If we know the speed and the centripetal acceleration, we can get the local radius, R. $R=V^2/A_{centripetal}$.

If we know the local radius, then we can know the length of that ten-degree segment. It will be 1/36th of the circumference, where "the circumference" is 2π times the radius. But what we really want, in order to calculate the location of the endpoint of the increment, is the length of the straight-line segment that cuts across the circular arc of the loop. The length of that segment is twice R times the sine of half the increment angle, i.e. 2*R*sin(10°/2), as shown in the diagram to the right (for only half the segment).



Once we know the length of that segment, and the angle off the horizontal (which is just θ at the midpoint, i.e. 5° for the 0°-to-10° segment, then 15° for the 10°-to-20°segment…), then we can calculate the coordinates of the start of the next segment. The x coordinate doesn't matter much, other than for plotting the shape of the loop later. But the y coordinate is crucially important: It tells us the change in height H during that segment.

Given the change in H, we can calculate how much kinetic energy $\frac{1}{2}mV^2$ became potential energy mgH, so we can calculate the speed at the end of this 10° segment, which will be the speed at the start of the next 10° segment. In other words, if the airplane goes up a small diagonal hill of a certain length and height, starting at a certain speed, we know how much speed it will lose by going up that small hill.

This looks like the following. Take an RV-3/4/6, which have a VNE around 200mph, and a limit load of 6g. To stay well away from these dangerous limits, can the airplane perform a loop pulling only 3g and going only at 160mph at the bottom? Let's try it…

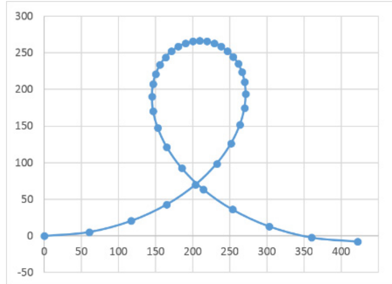
Cell Code.txt - Notepad

```
B16=B15+1
C16=B16*$D$12
D16=D15+J15
E16=E15+K15
F16=(1/$H$8)*SQRT((($H$8*$D$10)^2)-2*$H$10*E16)
G16=IF(F16>$D$11,$D$8,((F16)/$D$9)^2)
H16=(($H$8*(F16-0.5*(F15-F16)))^2)/($H$10*(G16-COS((C16+0.5*$D$12)*$H$9)))
I16=2*H16*SIN($D$12*$H$9/2)
J16=I16*COS(($C16+0.5*$D$12)*$H$9)
K16=I16*SIN(($C16+0.5*$D$12)*$H$9)
L16=(1/$H$8)*SQRT((($H$8*$D$10)^2)-2*$H$10*(E16+K16))
```

	Input parameters	
8	Limit Load (G)	3
9	Stall speed (mph)	65
10	Speed at bottom (e.g. VNE)	160
11	Maneuver speed	112.5833
12	Increment size (degrees)	10

	Constants and Conversions	
8	m/s in an mph	0.44704
9	radians in a degree	0.017453293
10	one G in m/s^2	9.81

	Segment (B)	Start angle (degrees) (C)	X at start (m) (D)	H at start (m) (E)	Start speed (mph) (F)	Gs (G)	Local Radius (m) (H)	Segment Length (m) (I)	Horizontal Component (m) (J)	Vertical Component (m) (K)	V at end (m/s) (L)
15	0	0	0	0	160	3	260.2604674	45.36639	45.19375589	3.953941302	158.7822945
16	1	10	45.19376	3.95394	158.782295	3	250.5671339	43.67673	42.18848087	11.30436938	155.2481787
17	2	20	87.38224	15.2583	155.248179	3	229.2032901	39.95277	36.20950292	16.88476851	149.8142667
18	3	30	123.5917	32.1431	149.814267	3	202.1193249	35.23172	28.86013527	20.20808427	143.0397201
19	4	40	152.4519	52.3512	143.03972	3	173.2754477	30.2039	21.357383	21.357383	135.5122678
20	5	50	173.8093	73.7085	135.512268	3	145.7299381	25.4024	14.57021921	20.80842952	127.7524251
21	6	60	188.3795	94.517	127.752425	3	121.28158	21.14077	8.934476471	19.16004662	120.1649761
22	7	70	197.314	113.677	120.164976	3	100.6415847	17.54298	4.5404584	16.94522144	113.0309868
23	8	80	201.8544	130.622	113.030987	3	83.80097324	14.60747	1.273125083	14.55188629	106.5239646
24	9	90	203.1275	145.174	106.523965	2.6858	78.34991879	13.65729	-1.190311318	13.60532062	100.0581646
25	10	100	201.9372	158.779	100.058165	2.3696	72.66136583	12.66571	-3.278127127	12.23413699	93.86445107
26	11	110	198.6591	171.014	93.8644511	2.0853	66.92158004	11.6652	-4.929926559	10.57226162	88.1622973
27	12	120	193.7292	181.586	88.1622973	1.8397	61.43771289	10.7093	-6.142601555	8.772544168	83.13444282
28	13	130	187.5866	190.358	83.1344428	1.6358	56.51404057	9.851046	-6.965741686	6.965741686	78.91429217
29	14	140	180.6208	197.324	78.9142922	1.474	52.40458798	9.134722	-7.482725854	5.239461049	75.58488681
30	15	150	173.1381	202.564	75.5848868	1.3522	49.28637518	8.591181	-7.786254487	3.630790095	73.18891943
31	16	160	165.3518	206.194	73.1889194	1.2678	47.26525829	8.238877	-7.958144446	2.132378377	71.74447096
32	17	170	157.3937	208.327	71.744471	1.2183	46.40237143	8.088466	-8.057687238	0.704956288	71.26050225
33	18	180	149.336	209.032	71.2605023	1.2019	46.74326997	8.147889	-8.116883648	-0.710135302	71.74801439
34	19	190	141.2191	208.322	71.7480144	1.2184	48.33595249	8.425512	-8.138419333	-2.180682888	73.22478226
35	20	200	133.0807	206.141	73.2247823	1.2691	51.22922577	8.929842	-8.093185745	-3.773914492	75.71246485
36	21	210	124.9875	202.367	75.7124649	1.3568	55.44568063	9.664819	-7.916956205	-5.543512414	79.22511179
37	22	220	117.0706	196.823	79.2251118	1.4856	60.9278838	10.62043	-7.509778025	-7.509778025	83.74902885
38	23	230	109.5608	189.314	83.7490289	1.6601	67.47026383	11.76084	-6.745741794	-9.633917698	89.21727018
39	24	240	102.8151	179.68	89.2172702	1.884	74.67454835	13.01663	-5.501066157	-11.79707444	95.48776509
40	25	250	97.31399	167.883	95.4877651	2.1581	81.98241452	14.29048	-3.698647471	-13.80354028	102.3381281
41	26	260	93.61534	154.079	102.338128	2.4788	88.80515097	15.47976	-1.349149787	-15.42085263	109.4854019
42	27	270	92.26619	138.658	109.485402	2.8372	94.68888898	16.50536	1.438536988	-16.44255302	116.6247001
43	28	280	93.70473	122.216	116.6247	3	107.3628807	18.71458	4.843690559	-18.07689926	124.0001694
44	29	290	98.54842	104.139	124.000169	3	128.8678345	22.46314	9.493334731	-20.35852203	131.8133696
45	30	300	108.0418	83.7803	131.81337	3	154.6481236	26.95694	15.46186796	-22.08183591	139.7951187
46	31	310	123.5036	61.6985	139.795119	3	183.6847475	32.01836	22.64040033	-22.64040033	147.5310943
47	32	320	146.144	39.0581	147.531094	3	214.1138975	37.32251	30.57281162	-21.40731316	154.489846
48	33	330	176.7168	17.6508	154.489846	3	242.8040247	42.32353	38.35814502	-17.88669678	160.0723602
49	34	340	215.075	-0.23591	160.07236	3	265.6471303	46.30535	44.7275295	-11.98470541	163.7063633
50	35	350	259.8025	-12.2206	163.706363	3	278.539539	48.55264	48.36788336	-4.231641473	164.9703606
51	36	360	308.1704	-16.4523	164.970361	3	278.8054631	48.59899	48.41406059	4.235681457	163.7051519
52	37	370	356.5844	-12.2166	163.705152	3	266.3291403	46.42423	44.84236087	12.01547438	160.0616852
53	38	380	401.4268	-0.2011	160.061685	3	243.6369805	42.46872	38.48973526	17.94805831	154.4592854
54	39	390	439.9165	17.747	154.459285	3	214.8471078	37.45032	30.67750497	21.48062023	147.4746931



Increment Size	Final Height
20°	-12.08 m
10°	-7.67 m
5°	-4.29 m
2°	-1.76 m
1°	-0.88 m
0.5°	-0.44 m
0.25°	-0.22 m
0.1°	-0.089 m

Apparently, despite being symmetric, the loop somehow ended up at a different height than where it started! I will not dive into the details, but this kind of error is inherent in numerical analysis. It builds up with each iteration, and is very hard to get rid of.

There are two things we will do to reduce the error.

One is the brute-force approach. For this particular set of inputs, this table shows the size of the final error as a function of how many degrees of looping are in each linear increment. As you can see, we really should use smaller increments. And why not? It just means copying and pasting more Excel rows.

The second way to reduce error: We know the speed at the start of each segment, and the speed at the end of each segment. However, the radius – and thus the segment length and its height – were calculated using the start speed only. So the whole segment was “too fast”, with too high a speed and too wide a radius. So let’s take our start speed and our “first iteration” end speed, take their average, and use this “average first-iteration” segment speed to come up with a “second iteration” radius, segment length, change in y (and also x) coordinate, and end speed. This second stage will tremendously improve accuracy, while only requiring less than twice the computational power.

Cell Code.txt - Notepad

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```
B16=B15+1
C16=B16*$D$12
D16=D15+O15
E16=E15+P15
F16=(1/$H$8)*SQRT((($H$8*$D$10)^2)-2*$H$10*E16)
G16=IF(F16>$D$11,$D$8,((F16)/$D$9)^2)
H16=(($H$8*(F16-0.5*(F15-F16)))^2)/($H$10*(G16-COS((C16+0.5*$D$12)*$H$9)))
I16=2*H16*SIN($D$12*$H$9/2)
J16=I16*COS(($C16+0.5*$D$12)*$H$9)
K16=I16*SIN(($C16+0.5*$D$12)*$H$9)
L16=(1/$H$8)*SQRT((($H$8*$D$10)^2)-2*$H$10*(E16+K16))
M16=(($H$8*AVERAGE(F16,L16))^2)/($H$10*(G16-COS((C16+0.5*$D$12)*$H$9)))
N16=2*M16*SIN($D$12*$H$9/2)
O16=N16*COS(($C16+0.5*$D$12)*$H$9)
P16=N16*SIN(($C16+0.5*$D$12)*$H$9)
```

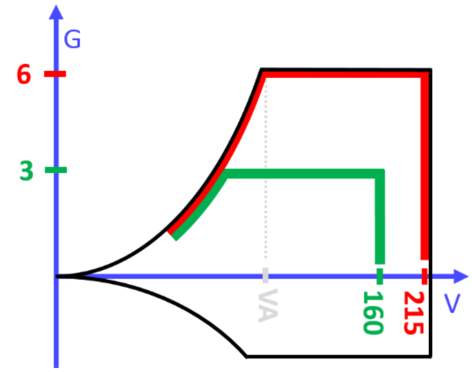
Implement this, and we get the following:

Input parameters	
Limit Load (G)	3
Stall speed (mph)	65
Speed at bottom (e.g. VNE)	160
Maneuver speed	112.583
Increment size (degrees)	2

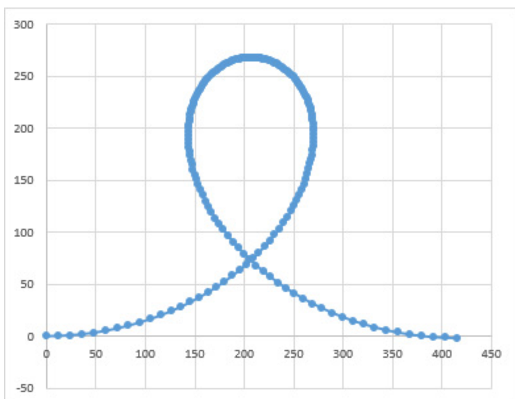
Constants and Conversions	
m/s in an mph	0.44704
radians in a degree	0.017453293
one G in m/s²	9.81

	Segment	Start angle (degrees)	X at start (m)	H at start (m)	Start speed (mph)	Gs	Local Radius (m)	Segment Length (m)	Horizontal Component (m)	Vertical Component (m)	1st iteration V at end (m/s)	2nd iteration Local Radius	2nd iteration Segment Length	2nd iteration Horizontal Component	2nd iteration Vertical Component
15	0	0	0	0	160	3	260.7357966	9.100934	9.099548073	0.158833202	159.9512624	260.65638	9.098162168	9.096776473	0.158784824
16	1	2	9.09678	0.15878	159.951277	3	260.3391003	9.087088	9.074634042	0.475581418	159.8052575	260.180738	9.081559976	9.069114014	0.475292126
17	2	4	18.1659	0.63408	159.805346	3	259.3905622	9.053979	9.019525912	0.789106268	159.5627691	259.233641	9.048501736	9.014069455	0.78862889
18	3	6	27.18	1.42271	159.562916	3	257.9776179	9.00466	8.937541105	1.09739206	159.2249552	257.823072	8.9992661	8.932186935	1.096734651
19	4	8	36.1121	2.51944	159.225158	3	256.1121332	8.939546	8.829485439	1.39845311	158.7934384	255.960859	8.934265891	8.824270252	1.397627105
20	5	10	44.9364	3.91707	158.793694	3	253.8095947	8.859176	8.696408385	1.69041055	158.2702723	253.662435	8.854039828	8.691366178	1.689430445
21	6	12	53.6278	5.6065	158.270576	3	251.0888342	8.764209	8.53958267	1.971518004	157.6579181	250.946567	8.759242972	8.534744142	1.970400942
22	7	14	62.1625	7.5769	157.658266	3	247.9717015	8.655406	8.360480038	2.240183875	156.9592179	247.83503	8.650635347	8.355872095	2.23894918
23	8	16	70.5184	9.81585	156.959604	3	244.482693	8.533623	8.160743926	2.494989802	156.1773628	244.352236	8.529069063	8.156389309	2.493658462
24	9	18	78.6748	12.3095	156.177781	3	240.6485481	8.399793	7.942159876	2.734704955	155.3158602	240.524834	8.395474308	7.93807691	2.733299076
25	10	20	86.6129	15.0428	155.316305	3	236.4978242	8.254912	7.706624544	2.958295994	154.3784965	236.381287	8.250844585	7.702827007	2.956838256
26	11	22	94.3157	17.9996	154.37896	3	232.060461	8.100027	7.456114136	3.164932678	153.3693001	231.951439	8.096221583	7.452611262	3.163445796
27	12	24	101.768	21.1631	153.369776	3	227.367345	7.936215	7.19265312	3.353989233	152.2925023	227.266082	7.932680052	7.189449703	3.352495454
28	13	26	108.958	24.5156	152.292984	3	222.4498839	7.764572	6.918283929	3.525041729	151.1524982	222.356529	7.76131302	6.915380537	3.523562377
29	14	28	115.873	28.0391	151.152979	3	217.3396003	7.586198	6.635038341	3.677861809	149.9538077	217.254213	7.583217668	6.632431616	3.676416878
30	15	30	122.506	31.7156	149.954281	3	212.0677514	7.402185	6.34491109	3.812407205	148.7010377	211.990311	7.399482121	6.342594116	3.811015027
31	16	32	128.848	35.5266	148.701497	3	206.6649828	7.213603	6.049836148	3.928809532	147.3988456	206.595389	7.211173379	6.047798873	3.92748651
32	17	34	134.896	39.4541	147.399286	3	201.161018	7.021488	5.751665995	4.027359887	146.0519041	201.099103	7.019326545	5.749895689	4.026120305
33	18	36	140.646	43.4802	146.052321	3	195.5843904	6.826837	5.452154089	4.108492796	144.6648684	195.529926	6.824935468	5.450635818	4.107348697
34	19	38	146.096	47.5875	144.665257	3	189.9622169	6.630596	5.15294062	4.172769038	143.2423466	189.914923	6.628944856	5.151657724	4.171730169
35	20	40	151.248	51.7593	143.242703	3	184.3200145	6.433656	4.855541529	4.220857856	141.7888714	184.279569	6.432243888	4.854476085	4.219931679
36	21	42	156.103	55.9792	141.789192	3	178.6815593	6.236846	4.561340696	4.253519012	140.3088758	178.647607	6.235661281	4.56047396	4.252710767
37	22	44	160.663	60.2319	140.309159	3	173.0687862	6.040934	4.271585112	4.271585112	138.8066712	173.040944	6.039961771	4.270897926	4.270897926
38	23	46	164.934	64.5028	138.806914	3	167.5017259	5.846616	3.987382796	4.275944546	137.2864284	167.479595	5.845843907	3.986855958	4.27537958
39	24	48	168.921	68.7782	137.28663	3	161.9984771	5.654527	3.709703183	4.267525342	135.7521619	161.981646	5.653939052	3.709317764	4.267081968
40	25	50	172.63	73.0453	135.752322	3	156.5752092	5.465228	3.439379661	4.247280164	134.2077164	156.563265	5.464811463	3.439117287	4.246956158
41	26	52	176.069	77.2922	134.207835	3	151.2461931	5.27922	3.177113948	4.216172612	132.6567568	151.238723	5.278959315	3.176957022	4.215964365
42	27	54	179.246	81.5082	132.656834	3	146.0238536	5.096935	2.923481978	4.175164959	131.1027602	146.020451	5.096816535	2.923413865	4.175067684
43	28	56	182.17	85.6833	131.102797	3	140.9188423	4.918746	2.678940978	4.125207351	129.5490104	140.919114	4.918755295	2.678946137	4.125215297
44	29	58	184.849	89.8085	129.549007	3	135.940125	4.744965	2.443837446	4.067228519	127.9985947	135.94369	4.745089047	2.443901528	4.06733517
45	30	60	187.293	93.8758	127.998554	3	131.0950809	4.575849	2.218415745	4.002127945	126.4544031	131.101576	4.576075961	2.218525649	4.002326217
46	31	62	189.511	97.8781	126.454326	3	126.3896102	4.411606	2.002827074	3.930769456	124.9191281	126.398691	4.411922655	2.002970971	3.93105187
47	32	64	191.514	101.809	124.919017	3	121.8282471	4.252392	1.797138587	3.853976136	123.3952677	121.839591	4.252788112	1.79730592	3.854334983
48	33	66	193.311	105.664	123.395125	3	117.4142749	4.098323	1.601342486	3.772526484	121.8851284	117.42758	4.098787692	1.60152394	3.772953964
49	34	68	194.913	109.436	121.884956	3	113.1498422	3.949474	1.415364923	3.687151684	120.3908303	113.164828	3.949997161	1.415552383	3.687640034
50	35	70	196.328	113.124	120.390631	3	109.0360768	3.805884	1.239074584	3.598533884	118.9143123	109.052487	3.806456663	1.239261071	3.599075482
51	36	72	197.568	116.723	118.914089	3	105.0731975	3.66756	1.072290856	3.507305354	117.4573394	105.090797	3.668174595	1.072470459	3.507892811
52	37	74	198.64	120.231	117.457094	3	101.2606202	3.534483	0.914791515	3.414048411	116.0215092	101.279194	3.535131317	0.914959312	3.414674638

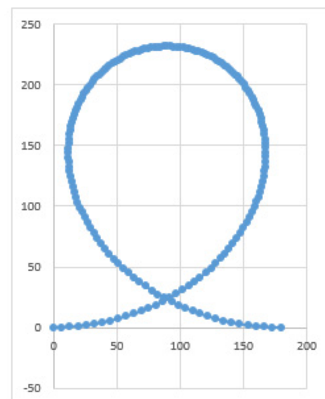
Here, we did the analysis using an entry speed of 160 mph, a stall speed of 65 mph, and the willingness to pull 3g. This leaves lots of margin, in an aerobatic airplane. What happens as we change the numbers?



Say we're ok with pulling 6g (because, for example, the airplane is being flown well under the max gross weight) and going to 215mph (a hair under the VNE for the RV-7/8/14). This means we go from flying in the "green" region to flying in the "red" region, as shown in the V-G envelope to the right. The result? The loop goes from "more teardropped" to "more circular":



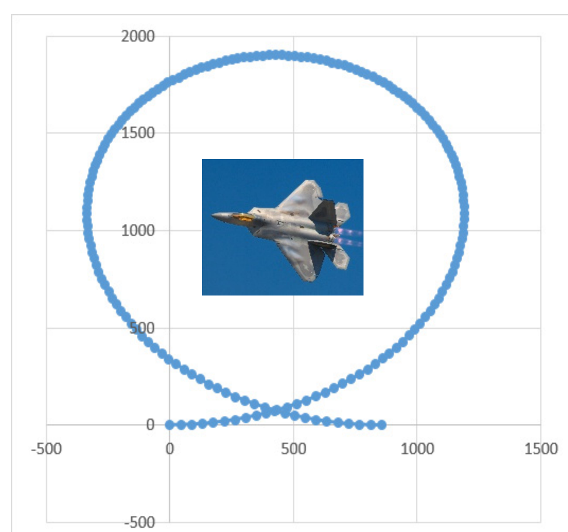
Flying an aerobatic RV with lots of margin



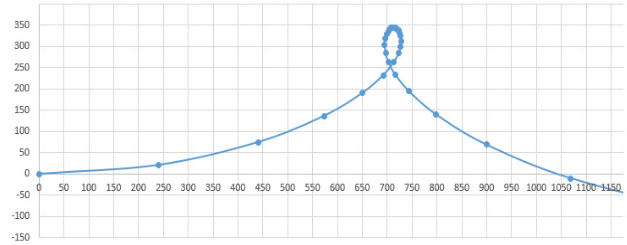
Flying an aerobatic RV at the edge of its V-G envelope

Interestingly, increasing the number of Gs (e.g. Say we're am Extra 300, or a Zivko Edge 540, and can pull 10g) only makes the loop slightly more circular, because fairly early on its way up the loop, the airplane's speed drops below the speed that would be needed to pull that many Gs (e.g. The speed needs to be three times the stall speed in order for the airplane to be aerodynamically capable of pulling 9g). So even if an airplane is structurally certified to safely pull 10g, during most of a loop, it is going too slowly to be aerodynamically able to do so without stalling.

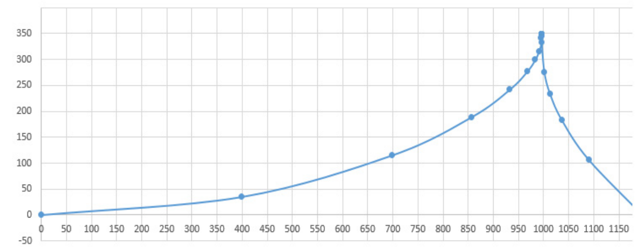
What has the greatest effect in loop "roundness" is increasing the ratio between entry speed and stall speed. The greater this ratio, the smaller a fraction of the airplane's speed is lost on its way up the loop, so the smaller the change in loop radius between the top and bottom, so the more circular the loop. For example, an F-22 can pull 10g, starts hitting CLmax at probably in the general vicinity of 200mph (that's a wild guess on my part), and would probably not want to start a loop any faster than 750mph (the speed of sound). Because it's going fast enough to pull about 10g the whole way around, and because it loses less than 20% of its speed as it goes around, the loop is pretty dang circular. (Recall that the axes are in meters. This is a mile-high loop!)



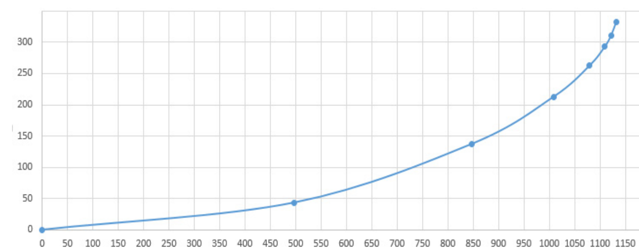
And what happens when the speed or G capability is too low? Gradually decrease the max speed and/or the allowable G limit, and you observe the following change:



If it has a little over the bare minimum speed and G capabilities, it makes a very narrow loop, almost like a backflip. (This assumes that it has elevator authority even at extremely slow speed, which is true for most single-engine propeller airplanes and for thrust-vectoring jet fighters, but not for airliners, business jets, etc.)

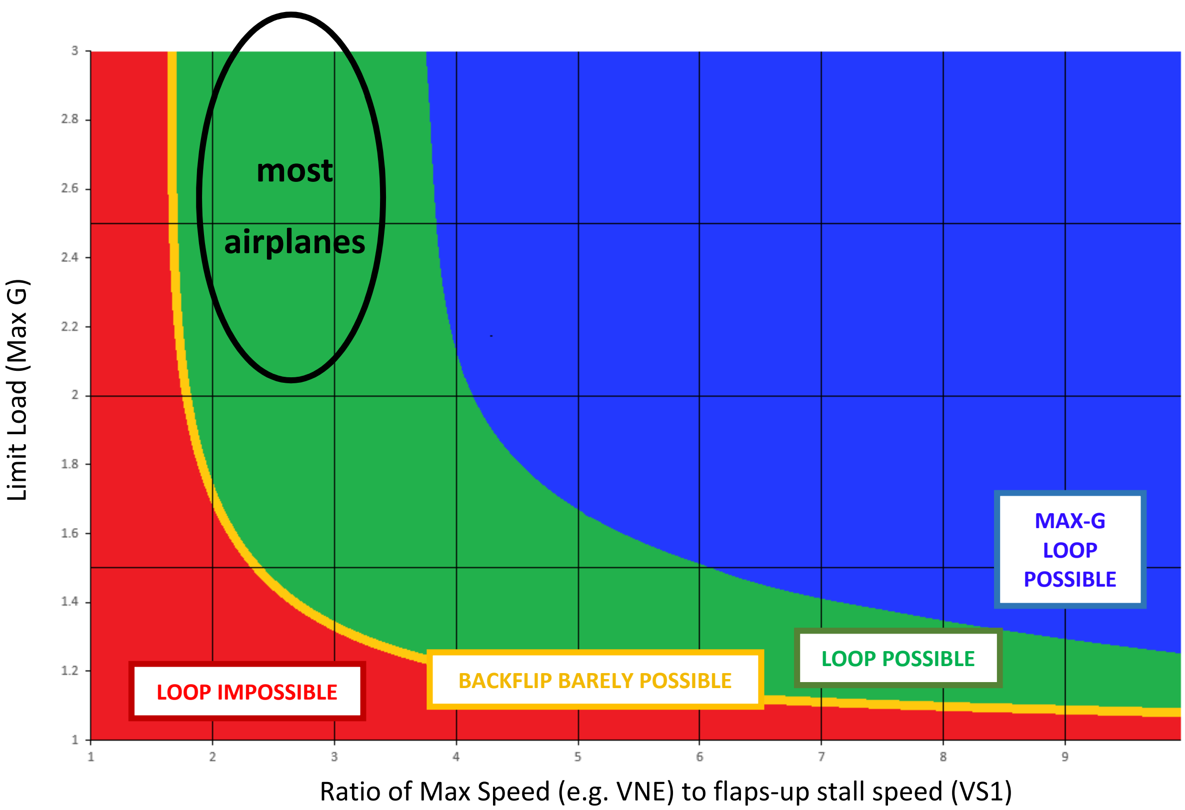


If it has the bare minimum speed and G capabilities, then its speed drops to zero just past 90° nose-up, and it falls on its back and performs a back-flip, which is basically a tail-slide. We can argue about whether this really “counts” as a “real” loop, but it makes for a neat dividing line: If an airplane has more than this combination of G capability and max-speed/stall-speed ratio, then it can do a loop.



If it has less than the bare minimum combination of G capability and max-speed/stall-speed ratio, then before it reaches the vertical, it approaches its stall speed and needs to stop pulling up. Either the nose comes down, or the airplane stalls. One way or another, the airplane can no longer follow the teardrop-shaped path that this analysis lays for it.

This numerical analysis was performed with many different combinations of max speed at the bottom, stall speed, and G capability. This yielded the following results:



In the graph on the previous page, the orange line shows combinations of G loads and speed ratios that would make an airplane "barely" able to do a loop. Airplanes with these G and speed capabilities will run out of airspeed when they reach 90 degrees nose-up. From there, they can simply fall backwards and perform a backflip on their way down. Again; While this is more of a tail-slide than a true loop, it is the bare minimum that could be considered a "loop". Any small additional capability in speed or Gs would results in a true loop where the pilot's butt is pushed down against the seat the whole time.

The green region shows combinations of G and speed capabilities that would allow an airplane to do a loop.

The blue region shows combinations of G and speed capabilities that would allow an airplane to do a loop while pulling limit load all the way around, i.e. without the speed ever dropping below VA.

The red region shows combinations of G and speed capabilities that would make it impossible for an airplane to do a loop, because it would stall before getting to 90 degrees nose-up.

So now we can answer the question:

Which airplanes can perform a loop? What are the bare minimum capabilities required?

The required capabilities are, very roughly and conservatively;
- Ability to pull about 2g (structurally), and
- Ability to safely fly at about twice the flaps-up 1g stall speed.

Or:
- Willingness to pull 1.5g, and
- Ability to safely fly at three times the flaps-up 1g stall speed.

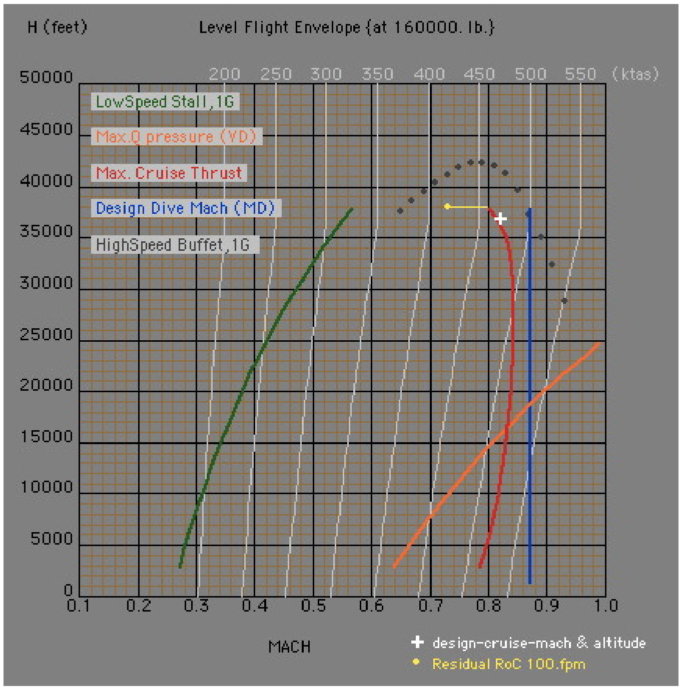
The g requirement is easy. Even the wimpiest airplane structures are certified to ~2.2g. Airliner structure is certified to 2.5g limit load, and most single-engine airplanes can do 3 or 4g. Aerobatic airplanes can do at least 6g.

But the "2x / 3x their stall speed" requirement is surprisingly tricky. Most single-engine airplanes do meet it, but not by much: Their VNE speed is often around three times their flaps-up stall speed, VS1. And that VNE is often only achievable in a dive: In level flight at max power, they are typically capable of 2 to 2.5 times their stall speed.

	Van's RV-6	Cirrus SR22	Beech A36 Bonanza	Piper J-3 Cub	Cessna 172	Lancair 4P	SportCruiser LSA	LearJet 23
VS1	65	70	68	38	57	69	32	104
VNE	185	201	205	121	182	274	138	561
Ratio	2.85	2.87	3.01	3.18	3.19	3.97	4.31	5.39

So could a single-engine airplane such as a Cub, Cessna, or LSA perform a loop? Definitely: Accelerate (dive if necessary) until flying at about 2.5 times the stall speed, then pull ~3g. Once the speed drops below $\sqrt{3}$ times the 1g stall speed, then pull a little less. The safest thing to do is as follows: Once the airplane has passed the vertical (90° nose-up), the pilot should pull just enough to keep their butt in the seat, i.e. just over zero g, thus preventing a stall by not requiring the wings to generate almost any lift. This is a similar rationale to the aileron roll. The airplane is almost in a ballistic parabola from 90° nose-up… until 90° nose-down, at which point the pilot most again pull about 3g and bring the nose back up to the horizon before the airplane picks up too much speed. (Some pilots will cut the engine power to idle at the top of the loop in order to reduce the risk of over-speeding on the way down).

Interestingly, excluding airshow aerobatic airplanes and jet fighters, it is business jets that offer the greatest margins when it comes to performing loops. Their combination of high max speeds (very close to those of jetliners) and slow landing speeds (to allow operations from small airports) means that they should be able to easily reach the top of a loop with plenty of speed left over to avoid a stall. (If you know of a Gulfstream 6 that I could borrow to verify this, please let me know).



Could an airliner do a loop? Looking at the A320 flight envelope to the left, a loop seems doable: The max speed is more than twice the stall speed (at most altitudes), and the airplane can pull 2.5g, right?

However, there are two problems.

One problem is that it is difficult to get flaps-up stall speeds for airliners. Most stall data, such as what is shown to the left, is for flaps-down performance. (And it would be impractical to expect a pilot to deploy the flaps on the way up a loop, and retract them again on the way down). In addition, most published numbers for airliner slow-speed flight, such as approach speeds, have an unclear amount of margin or "padding" between these recommended minimum speeds and the speeds at which the airplane will actually not be able to sustain 1g flight.

The second problem is that an airplane that enters a loop at over 500mph but will only pull up at less than 2.5g will end up doing a very tall loop, about 2 miles i.e. up 10,000 feet or so. This means that the stall speed at the top will be higher: At altitude, fewer Gs can be pulled at the airplane's speed at any given time before it stalls. Unfortunately, when determining the ratio between maximum speed and minimum speed, what counts is the maximum speed at the bottom of the loop (which will be relatively low, i.e. less energy to work with) and the stall speed at the top of the loop (which will be relatively high, i.e. less G capability and a wider loop), making for a less favorable ratio.

The good news is that airliner numbers like these nearly always (conservatively) correspond with performance at max gross weight. When flown nearly empty, an airliner's weight is reduced by almost half, which allows the stall speed to be reduced by nearly a third. So even if a full airliner cannot quite perform a loop, an empty airliner should be able to.

Let us perform our numerical analysis one more time, and "sharpen our pencil" even further: We will vary the stall speed as a function of altitude. The higher the airplane flies, the thinner the air will be, so the higher the stall speed will be, and the fewer Gs can be pulled (at a given speed) before a stall. The stall speed will rise as the square root of the drop in density (e.g. if density drops to ¼ of sea-level density, the stall speed goes up by two, in order to provide the same dynamic pressure $\frac{1}{2}\rho v^2$). The drop in density will be given by NASA's standard atmospheric model, https://www.grc.nasa.gov/www/k-12/airplane/atmos.html

Conservatively, we will say that the max speed is 550 mph (~478 knots), which is lower than the true max speed until about 30,000 feet. Conservatively, will only allow the airplane to pull 2.2g, even though it could pull just over 2.5g when full and probably over 4g if nearly empty. And conservatively, we will estimate the sea-level flaps-up stall speed at 220mph, based on the stall speeds of airplanes with similar airfoils, adjusted by differences in the flaps-up wing loading (weight divided by wing area), then multiplied by about 1.2 to be safe (i.e. just add 20% and call it an airliner with an extra fast stall speed. Even the Concorde landed at 185mph with no flaps or slats, although one wonders whether a conventional airliner could match the Concorde's high-alpha capabilities). This is a lot of hand-waving, but it is conservative, i.e. estimating the worst case for each step. If our crudely-modeled airliner (which can only go 550mph, can only pull 2.2g, and stalls at a very high speed of 220mph) can do a loop, then a real airliner certainly can.

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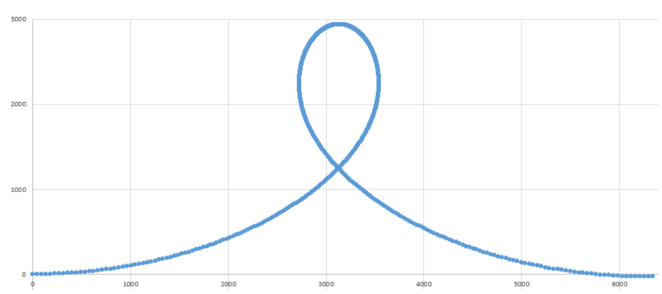
```
B11=B10+1
C11=B11*$D$6
D11=D10+O10
E11=E10+P10
F11=(1/$H$3)*SQRT((($H$3*$D$5)^2)-2*$H$5*E11)
G11=IF(F11>S11,$D$3,(F11/R11)^2)
H11=(($H$3*(F11-0.5*(F10-F11)))^2)/($H$5*(G11-COS((C11+0.5*$D$6)*$H$4)))
I11=2*H11*SIN($D$6*$H$4/2)
J11=I11*COS(($C11+0.5*$D$6)*$H$4)
K11=I11*SIN(($C11+0.5*$D$6)*$H$4)
L11=(1/$H$3)*SQRT((($H$3*$D$5)^2)-2*$H$5*(E11+K11))
M11=(($H$3*AVERAGE(F11,L11))^2)/($H$5*(G11-COS((C11+0.5*$D$6)*$H$4)))
N11=2*M11*SIN($D$6*$H$4/2)
O11=N11*COS(($C11+0.5*$D$6)*$H$4)
P11=N11*SIN(($C11+0.5*$D$6)*$H$4)
Q11=(2116*(((59-0.00356*(E11+$H$7*$H$6)/$H$6)+459.7)/518.6)^5.256)/(1718*((59-0.00356*(E11+$H$7*$H$6)/$H$6)+459.7))
R11=$D$4/SQRT(Q11/0.00237692987919529)
S11=R11*SQRT($D$3)
```

Input parameters	
Limit Load (G)	2.2
Stall speed at sea level	220
Speed at bottom (e.g. VNE)	550
Increment size (degrees)	0.5

Constants and Conversions	
m/s in an mph	0.44704
radians in a degree	0.017453293
one G in m/s²	9.81
meters in one foot	0.3048
starting altitude (ft)	0

Segment	Start angle (degrees)	X at start (m)	H at start (m)	Start speed (mph)	Gs	Local Radius (m)	Segment Length (m)	Horizontal Component (m)	Vertical Component (m)	1st iteration V at end (m/s)	2nd iteration Local Radius	2nd iteration Segment Length	2nd iteration Horizontal Component	2nd iteration Vertical Component	Density	Stall Speed	Maneuver Speed
0	0	0	0	550	2.2	5135.283795	44.8137	44.81323633	0.195535872	549.982548	5135.1208	44.81224095	44.81181437	0.195529667	0.0023769	220	326.3127
1	0.5	44.8118	0.19553	549.98255	2.2	5134.469142	44.8066	44.80271508	0.586499668	549.930199	5134.1433	44.80371063	44.79987218	0.586462452	0.0023769	220	326.3158
2	1	89.6117	0.78199	549.9302	2.2	5132.514694	44.7895	44.77883935	0.977077751	549.842979	5132.1892	44.78665739	44.77599938	0.977015782	0.0023768	220.01	326.325
3	1.5	134.388	1.75901	549.84298	2.2	5129.585031	44.7639	44.74305368	1.367023928	549.720927	5129.26	44.76109548	44.74021849	1.366937306	0.0023765	220.02	326.3402
4	2	179.128	3.12595	549.72094	2.2	5125.682602	44.7299	44.69539186	1.756086729	549.564101	5125.3582	44.72704626	44.69256333	1.755975596	0.0023762	220.03	326.3616
5	2.5	223.82	4.88192	549.56411	2.2	5120.810668	44.6874	44.6358989	2.144015972	549.37257	5120.4871	44.68453818	44.63307887	2.143880516	0.0023758	220.05	326.3891
6	3	268.454	7.0258	549.37258	2.2	5114.973294	44.6364	44.56463086	2.530563129	549.146422	5114.6508	44.63360671	44.56182116	2.530403583	0.0023753	220.07	326.4226
7	3.5	313.015	9.5562	549.14644	2.2	5108.175342	44.5771	44.48165481	2.915481688	548.88576	5107.8541	44.57429424	44.47885725	2.915298326	0.0023748	220.1	326.4622
8	4	357.494	12.4715	548.88578	2.2	5100.422461	44.5094	44.38704864	3.298527499	548.590702	5100.1026	44.50665007	44.38426502	3.29832064	0.0023741	220.13	326.5078
9	4.5	401.878	15.7698	548.59072	2.2	5091.72108	44.4335	44.28090096	3.679459128	548.261382	5091.4028	44.43073028	44.27813305	3.679229133	0.0023733	220.17	326.5594
10	5	446.157	19.4491	548.2614	2.2	5082.078394	44.3494	44.16331091	4.058038198	547.897949	5081.7619	44.34659763	44.16056044	4.057785465	0.0023725	220.21	326.617
11	5.5	490.317	23.5068	547.89797	2.2	5071.502352	44.2571	44.03438797	4.434029715	547.500568	5071.1878	44.25432146	44.03165665	4.433754686	0.0023716	220.25	326.6806
12	6	534.349	27.9406	547.50059	2.2	5060.001643	44.1567	43.89425178	4.8072024	547.069416	5059.6892	44.15397759	43.89154128	4.806905553	0.0023706	220.29	326.75
13	6.5	578.24	32.7475	547.06944	2.2	5047.585681	44.0484	43.74303187	5.177329002	546.604687	5047.2755	44.04564813	43.74034383	5.177010851	0.0023695	220.35	326.8254
14	7	621.981	37.9245	546.60472	2.2	5034.264591	43.9321	43.58086746	5.544186598	546.106591	5033.9569	43.9294214	43.57820346	5.543847695	0.0023683	220.4	326.9065
15	7.5	665.559	43.4684	546.10662	2.2	5020.049185	43.8081	43.40790717	5.907556894	545.575348	5019.7441	43.80539172	43.40526878	5.907197825	0.002367	220.46	326.9934
16	8	708.964	49.3756	545.57538	2.2	5004.95095	43.6763	43.22430879	6.267226505	545.011195	5004.6486	43.67365932	43.22169751	6.266847887	0.0023657	220.52	327.0861
17	8.5	752.186	55.6424	545.01123	2.2	4988.982026	43.5369	43.03023894	6.622987223	544.414383	4988.6826	43.53433007	43.02765623	6.622589706	0.0023643	220.59	327.1845
18	9	795.214	62.265	544.41442	2.2	4972.155184	43.3901	42.82587278	6.974636279	543.785175	4971.8588	43.3875154	42.82332006	6.974220541	0.0023628	220.66	327.2884
19	9.5	838.037	69.2392	543.78521	2.2	4954.483807	43.2359	42.61139374	7.321976586	543.123847	4954.1906	43.23333205	42.60887236	7.321543332	0.0023612	220.73	327.398
20	10	880.646	76.5608	543.12389	2.2	4935.981864	43.0744	42.38699312	7.66481697	542.43069	4935.692	43.07190189	42.38450438	7.664366932	0.0023595	220.81	327.5131
21	10.5	923.03	84.2251	542.43073	2.2	4916.663891	42.9059	42.1528698	8.002972387	541.706005	4916.3776	42.90335171	42.15041496	8.00250632	0.0023578	220.89	327.6336
22	11	965.181	92.2276	541.70605	2.2	4896.544964	42.7303	41.90922986	8.336264126	540.950107	4896.2622	42.72781305	41.90681011	8.335782808	0.002356	220.98	327.7595
23	11.5	1007.09	100.563	540.95015	2.2	4875.640677	42.5479	41.65628624	8.66452	540.163324	4875.3617	42.54542193	41.65390272	8.664024226	0.0023541	221.06	327.8907
24	12	1048.74	109.227	540.16337	2.2	4853.967114	42.3587	41.39425838	8.987574511	539.345992	4853.692	42.35631867	41.39191216	8.987065097	0.0023521	221.16	328.0272
25	12.5	1090.13	118.215	539.34604	2.2	4831.540825	42.163	41.1233718	9.305269017	538.498461	4831.2697	42.16064764	41.1210639	9.304746793	0.0023501	221.25	328.1689
26	13	1131.25	127.519	538.49851	2.2	4808.378799	41.9609	40.84385777	9.617451868	537.621093	4808.1117	41.95855705	40.84158915	9.616917678	0.002348	221.35	328.3156
27	13.5	1172.1	137.136	537.62114	2.2	4784.498438	41.7525	40.55595289	9.923978536	536.714257	4784.2355	41.75019872	40.55372444	9.923433237	0.0023458	221.45	328.4674
28	14	1212.65	147.06	536.71431	2.2	4759.917531	41.538	40.25989871	10.22471172	535.778334	4759.6589	41.53572782	40.25771126	10.22415618	0.0023436	221.56	328.6241
29	14.5	1252.91	157.284	535.77839	2.2	4734.654224	41.3175	39.95594134	10.51952146	534.813716	4734.4	41.31530266	39.95379565	10.51895655	0.0023413	221.67	328.7857
30	15	1292.86	167.803	534.81377	2.2	4708.726995	41.0913	39.64433105	10.80828519	533.820804	4708.4772	41.08908445	39.64222783	10.80771178	0.0023389	221.78	328.9521
31	15.5	1332.5	178.61	533.82086	2.2	4682.154627	40.8594	39.32532184	11.09088781	532.800005	4681.9093	40.85723704	39.32326173	11.0903068	0.0023365	221.89	329.1231
32	16	1371.83	189.701	532.80006	2.2	4654.956177	40.622	38.99917112	11.36722177	531.751739	4654.7155	40.61992668	38.9971547	11.36663403	0.002334	222.01	329.2988
33	16.5	1410.82	201.067	531.75179	2.2	4627.150954	40.3794	38.66613923	11.63718704	530.676431	4626.9149	40.37732183	38.66416701	11.63659347	0.0023315	222.13	329.479
34	17	1449.49	212.704	530.67649	2.2	4598.758485	40.1316	38.32648909	11.90069119	529.574516	4598.5272	40.12959283	38.32456152	11.90009266	0.0023289	222.26	329.6636
35	17.5	1487.81	224.604	529.57457	2.2	4569.798493	39.8789	37.98048579	12.15764935	528.446435	4569.572	39.87691175	37.97860326	12.15704675	0.0023262	222.39	329.8526
36	18	1525.79	236.761	528.44649	2.2	4540.290867	39.6214	37.62839622	12.40798424	527.292637	4540.0692	39.61945211	37.62655905	12.40737843	0.0023235	222.52	330.0458
37	18.5	1563.42	249.168	527.29269	2.2	4510.255637	39.3593	37.27048867	12.65162611	526.113577	4510.0388	39.35738864	37.26869712	12.65101796	0.0023207	222.65	330.2432
38	19	1600.69	261.819	526.11363	2.2	4479.712944	39.0927	36.90703243	12.88851271	524.909717	4479.5011	39.09089707	36.90528672	12.88790308	0.0023179	222.79	330.4446
39	19.5	1637.59	274.707	524.90977	2.2	4448.683018	38.822	36.53829746	13.11858925	523.681523	4448.4761	38.82015387	36.53659773	13.11797899	0.002315	222.92	330.65
40	20	1674.13	287.825	523.68158	2.2	4417.186149	38.5471	36.164554	13.34180835	522.429468	4416.9842	38.54533606	36.16290033	13.34119828	0.0023121	223.07	330.8592

As we can see, the airliner does go all the way around. At the top of the loop, the airplane is flying at about 118mph (and the stall speed is 254), so the wings can only pull about 0.2g (i.e. the airplane is almost on a weightless ballistic parabola over the top of the loop).



The only problem here is that a speed of 550mph is above an airliner's max-thrust level-flight speed at sea level. To correct for this, we can either…

(A) … enter a lower speed at the start of the maneuver (i.e. at the bottom of the loop), or…

(B) … start the loop at about 15,000 to 20,000 feet where the airplane starts being able to reach 500mph in level flight, or…

(C) … require the airplane to start the maneuver by diving to pick up speed.

Trying option (A) reveals that, if we restrict ourselves to pulling 2.2g and stick with our estimated flaps-up sea-level 1g stall speed of 220mph, then the airplane must start the loop while going at 415mph or more, in order to make it all the way around the top. This is probably attainable at sea level by most airliners, based on the A320 envelope on the previous page, where the airplane can apparently reach a max Q (orange line) corresponding to about 400kts (460mph) at sea level. It would depend how over-powered the airliner is.

Trying option (B) shows that, starting at 17,000 feet (the altitude where the A320's envelope seems capable of reaching the fastest speeds), the airplane can barely perform a loop if starting at 550mph and sticking with our 2.2g, 220mph-stall restrictions.

Of course, pulling the full 2.5g, and/or lowering our guessed flaps-up sea-level 1g stall speed from 220 to a more reasonable 180 or even 200, shows that a loop is perfectly possible, whether starting at sea level around 370mph or starting at 17,000 feet at 550mph.

The ability of an airliner to perform a loop is therefore largely dependent on its low-altitude speed performance. Can it dive to 550mph at just above sea level without coming apart? How fast can it go in level flight just above sea level? At what altitude can it reach 550mph in level flight? And, just as importantly: What is its flaps-up stall speed? The answer will be different for different airliners, some of which are more overpowered than others, and some of which have higher wing loading than others.

One the one hand, we can conclude that airliners are generally capable of loops. This is especially true of airliners flown at well below their max weight, which are capable of pulling 2.5g or more without damage, and would certainly stall (flaps-up, at sea level, doing 1g) at well under 220mph.

On the other hand, flying an airliner through a loop requires pulling a very precise amount of Gs. While flying faster than the maneuver speed (which changes with altitude), the pilot must pull nearly the limit load, just under 2.5g. Then, once flying slower than the maneuver speed (which would happen starting a few thousand feet into the loop), the pilot must pull the square of the ratio between the current speed and the current stall speed, i.e. maintain the angle of attack at just below the value that would cause the airplane to stall. This is nearly impossible to do in practice, without very accurate live information about the g level and the angle of attack, as well as (ideally) the maneuver speed (continuously adjusted for local air density).

If the pilot pulls slightly too many Gs, at low altitude the wings would be damaged, and at high altitudes the wings would stall. If the pilot pulls slightly too few Gs, then the airplane would run out of airspeed just before reaching 90 degrees nose-up, and risks being put into a tail-slide (which could damage the control surfaces) or maybe even an inverted spin (from which it may or may not be possible to recover).

It would arguably be easier – and less dangerous – to program this required level of G-pulling into the fly-by-wire system as a special pitch law, rather than trying to fly it by hand…

Another way to ask the question: If exactly the max safe Gs are pulled and the airliner performs a loop, one way to quantify the precision required (i.e. the margin) would be to ask: How much fewer Gs could still be pulled with the airplane still performing a loop? 10% less Gs? Maybe 0.5g less?

Conservatively assuming a flaps-up sea-level 1g stall speed of 220mph, and using a max sea-level speed of 460mph, pulling 2.5g gives us a nice loop. The bare minimum number of Gs that could be pulled and still give us a backflip is 15% less, i.e. pulling 2.125g at the start but still nailing the ~0.1g at the top of the loop. Looking at an absolute (subtractive) rather than multiplicative error, we can only be off throughout the whole loop by 0.15g.

Can a human pilot fly an entire loop while pulling a number of G to within 0.15g? Probably not safely.

If the stall speed is a more reasonable 180mph, the required precision is a more doable range of ±0.4g.

The more speed we have going in (e.g. 550mph rather than 460mph), the more Gs we can pull (e.g. 3g instead of 2.5), and the lower the stall speed (e.g. 180mph rather than 220mph), the less precise the loop can be, i.e. the greater the range of g values that are enough to bring the airplane all the way around without falling off before reaching the top, but not enough to stall or break the wings.

Exercises for the reader

(A.k.a. I have not gotten around to doing this yet but it would be fun, meaningful, and educational. Hopefully soon).

I) The last topic covered in the previous page shows that, if the G capability of an airplane is 2.5 and the ratio between max speed and stall speed is 460/220, a backflip is barely possible if the pilot pulls the highest Gs allowable (from a structural point of view and a stall point of view, whichever is less), ***minus 0.15g***. We then saw that, if the G capability of an airplane is 2.5 and the ratio between max speed and stall speed is 460/180 (i.e. a little better than 460/220), a backflip is barely possible if the pilot pulls the highest Gs allowable (from a structural point of view and a stall point of view, whichever is less), ***minus 0.4g*** (which is easier to do than to within 0.15g).

In general, what is the relationship between the limit load, the speed ratio, and the precision required (to within how many Gs / to withing what fraction of a g) to fly a loop? Rather than just "pass/fail" (the boundary shown by the orange line in the loop capability graph), what airplanes can perform loops easily and forgivingly, and which airplanes require robot precision? Here is one way to quantify this:

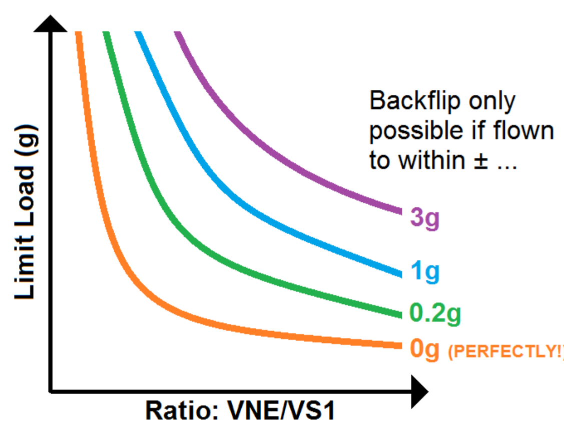
The orange line on the G/speed-ratio chart shows the combinations of G capability and max-speed/stall-speed ratios where an airplane is barely able to perform a backflip, if flown perfectly, if the pilot pulls maximum Gs the whole time. By that I mean: If the pilot pulls exactly the limit load while faster than the maneuver speed, and the square of the ratio between the speed and the stall speed while slower than the maneuver speed, a backflip will be barely possible.

For what combinations of G capability and speed ratio (i.e. imagine a line in the geen zone, parallel to the orange line) is a backflip barely possible if the pilot pulls 0.1g less than the maximum Gs? (Where "maximum Gs" is limit load when above maneuver speed, and the square of the ratio between the speed and the stall speed when below maneuver speed).

For what combinations of G capability and speed ratio (i.e. imagine another line in the geen zone, parallel to the previous one) is a backflip barely possible if the pilot pulls 0.2g less than the maximum Gs?

For what combinations of G capability and speed ratio (i.e. imagine yet another line in the geen zone, parallel to the previous ones) is a backflip barely possible if the pilot pulls 1g less than the maximum Gs?

The graph could be filled with these "precision lines", and would end up looking something like this:



All that is really required to create this graph is to take the original orange line and move it over. Moving it upwards by 1g shows the combinations of speed ratio and g capability that allow an airplane to be flown through a loop (or, barely, at least through a backflip) with "1g to spare", i.e. while remaining 1g below limit load. Moving the orange line to the right by 0.5 would show the combinations of speed ratio and g capability that allow an airplane to be flown through a loop (or, barely, at least through a backflip) with "0.5 VS1 to spare", i.e. while remaining below the max safe speed by a margin of "half a stall-speed".

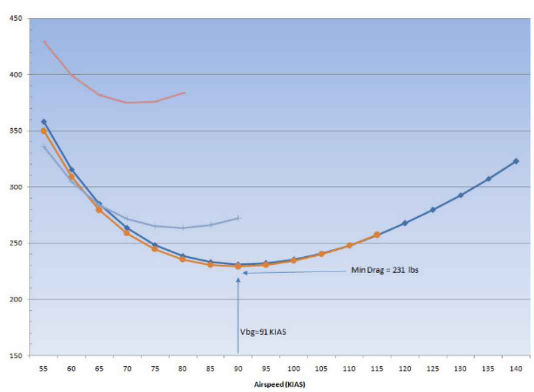
II) On page 4, we list four ways to model a loop, each more accurate than the previous… but we never get to number four. Throughout this paper, all the math assumes that drag and thrust are perfectly balanced out, so the changes in speed are due only to vertical travel, i.e. to "going uphill" or "going downhill".

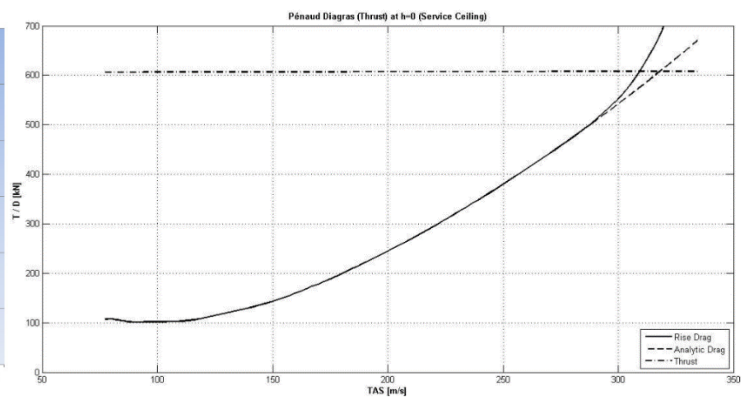
How would we model the fact that, as the airplane gets slower on its way up, there is less drag, and more excess thrust? What would the impact be to the loop?

II A) We can assume that the airplane is in steady level flight at the beginning of the loop, i.e. that thrust and drag are equal and opposite, that a dive was not necessary to pick up enough speed for a loop. (This includes the increase in induced drag that happens when the airplane starts pulling so-many Gs to get into the loop).

In this scenario, as soon as the airplane starts going uphill and losing speed, it will have "excess thrust" (i.e. some thrust in addition to what is needed to overcome drag). That will be the case even more so once the airplane is below maneuver speed and starts pulling fewer Gs.

Over the time of each analysis segment (10 degrees or 2 degrees or whatever), how much speed will this thrust add? That depends on the airplane's power curve, which can be looked up for many airplanes. Below is one for the RV-10 and one for the A340.





Sources: http://www.azcloudflyer.com/flight_test.html and http://www.davidegenovese.com/Progetti_files/Elaborato%20A340_300%20-%20J1.pdf

Subtract the value of the curve at each speed (and don't forget to modify the values to take into account the extra induced drag from the Gs being pulled) from the available thrust, and that gives the excess thrust. Divide it by the airplane's weight, and you get the acceleration that it provides, i.e. how much additional speed per second (because F=ma) you get from the engines. Multiply this "additional speed per second" by the duration of the segment to get the extra speed in that segment. Add this "additional speed from having a little excess thrust for a short period of time" to the "speed at the end of this segment" at each segment of the analysis, and…

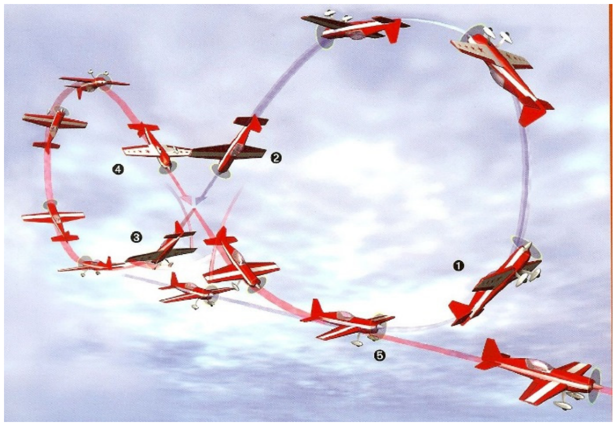
… You will find that the airplane is faster all the way around the loop than it was when we were neglecting the excess thrust. And this should be no surprise!

This means the airplane will end the loop higher, and faster, than it started, because of all the extra energy that it picked up on the way around the loop.

II B) This all changes, of course, if a dive was necessary before the start of the loop to pick up speed. This means that at the start of the loop, the airplane has excess drag, rather than excess thrust, and will slow down more quickly than it did when we were neglecting all this. Once the airplane slows past its terminal velocity (terminal velocity while pulling this-many Gs at this engine setting) and has some excess thrust… will this give the airplane enough energy, for the little while when it has some excess thrust, to finish the loop at the same speed and altitude as when it started? That will depend on how underpowered/overpowered the airplane is.

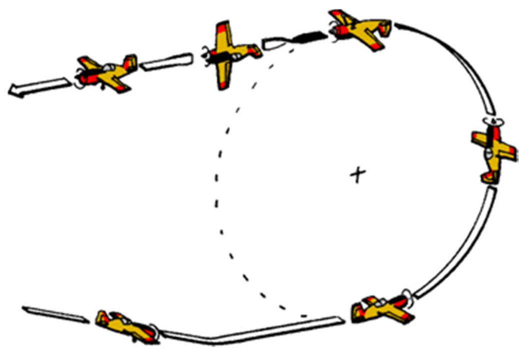
III) What about aerobatics that are made of pieces of loops and rolls, such as Immelmans, Split-Ss, and Cuban 8s? Can a given airplane perform those? How much more speed and G capability does an airplane need (if any) in addition to what is needed to perform a loop?

A Cuban 8 requires the airplane to roll 180 degrees while it's about ¼ of the way down from a loop. This means that it needs enough speed not only to barely make it over the loop, but enough speed to be only 45 degrees down from the top of the loop (at which point it has not picked up a whole lot of speed since the top of the loop) and then roll halfway around without taking too long. If it takes too long, it will go downhill for too long, pick up too much speed, and over-speed at the bottom. The faster the roll rate (which, of course, is a function of speed…), the less downwards the airplane has to point before it rolls back upright. An airplane with a very slow roll rate might only be able to safely accomplish a Cuban 8 if it starts the loop at well below max speed, so that it can end the Cuban 8 lower than the start of the first loop without over-speeding. This means it needs enough speed capability to start a loop at less than its max safe speed and still have enough speed to make it over the top… So either you need powerful ailerons, or a high VNE/VS1 ratio.



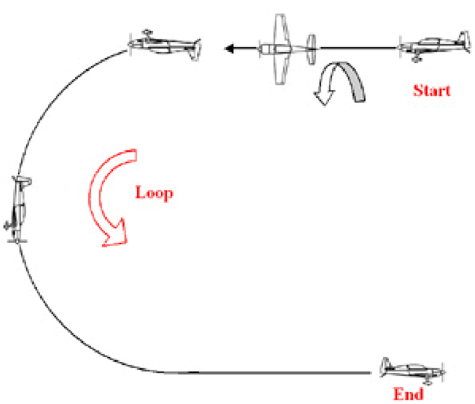
CUBAN 8

An Immelman is the same as the first half of the loop, with the additional requirement that the speed at the top must be enough for 1g level flight (and the aileron capability must be enough to roll the airplane around once it's getting up to the top at this speed). In short, this requires either more G capability than a loop does, or a higher VNE/VS1 ratio. Everything else being equal, if an airplane (with a given VNE/VS1 ratio) needs to pull so many Gs in order to barely make it around a loop, then it needs one more G in order to perform an Immelman. (The difference between the speed-&-G requirement for a loop and the speed-&-G requirement for an iImmelman is similar to the difference between the speed-&-G requirement for a rigid circular loop and the speed-&-G requirement for a non-rigid circular loop).



IMMELMAN

A Split-S is the same as an Immelman, but backwards. The airplane must be flying close to stall speed, then roll inverted, then pull up into the second half of a loop. Given the VS1/VNE ratio, how many Gs must it pull in order to not over-speed? About one more G than is required for a loop. The added complication, as with the Immelman, is to have enough aileron authority at the slow end near the top to roll around. When flying slowly and trying to roll as much as possible, it is easy to enter a snap roll (i.e. to stall one wing, causing the non-stalled wing to roll the airplane in the direction opposite of what was commanded) by mistake! For both the Split-S and the Immelman, the maneuver is made easier by having a slightly diagonal segment at the top, i.e. by starting the Split-S with a slight climb (similar to how one starts an aileron roll) or completing the Immelman with a slight descent while/after rolling (until the airplane has enough speed for 1g flight).



SPLIT "S"

IV) Can a given airplane sustain inverted (-1g) flight? How about knife-edge flight?

While all previous topics on this paper have primarily depended on kinetic energy being temporarily exchanged for gravitational potential energy, the question of whether an airplane can sustain -1g inverted flight and/or sideways knife-edge flight is fundamentally a stability-and-control question, related to the stabilizing effect (i.e. size and position) of the tail fins, and to the capability (i.e. force) that the control surfaces can exert. At the high angles of attack required for inverted and especially knife-edge flight, the tail fins will try to lift the tail up in the air and cause the nose to point below the horizon. The control surfaces will “fight” this force and try to keep the nose high above the horizon. Who will win?

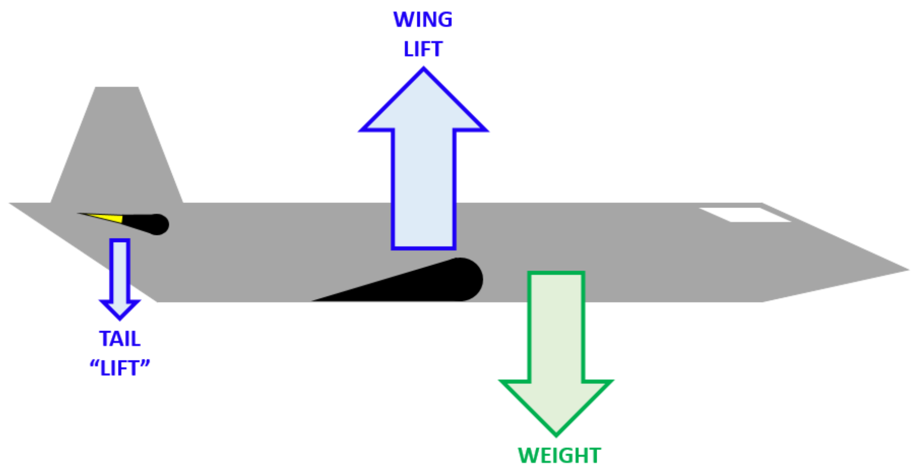
If the tail fins do not have a very large stabilizing effect (i.e. if the center of gravity is close to the center of lift, requiring small forces from the tail fins), and if the control surfaces are relatively large, then the airplane should be able to sustain inverted and knife-edge flight, assuming the structure and engines and systems are up for it.

(Most airliners are certified to -1g, at least so that they can survive the occasional turbulence that is rough enough to send stuff flying towards the ceiling. However, most piston-powered airplanes have fuel pumps and oil pumps that will stop working if the airplane is inverted for more than a second or two. But let’s disregard this for now and assume that the structure, the fuel systems and engine, and other components can take -1g flight. So the question becomes, aerodynamically, can the airplane sustain -1g?)

As with loops and rolls, speed helps: The faster an airplane flies, the less negative the angle of incidence of the horizontal stabilizer has to be, and the more force is generated by the elevators at any given angle of deflection. But, given realistic flying speeds for a Cessna or a 747: Are their horizontal stabilizers small enough, and their elevators big enough, to sustain inverted flight?

Why is that (previous sentence) the best way to phrase the question? Below is an introduction to the physics of stability and balance in an airplane, which are what determines whether an airplane can sustain inverted flight. (Knife-edge flight, as you will see, is a slight twist on this).

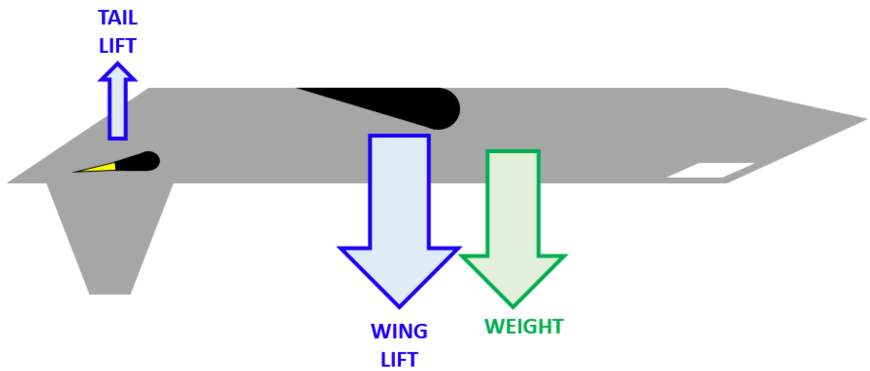
Nearly all airplanes fly with the center of gravity ahead of the center of lift, and the horizontal stabilizer at a slightly negative angle of attack. This means that the weight is closer to the front than to the back, making the nose want to come down, but the horizontal tail fins “push down” and the airplane is balanced around the wings like a see-saw.



For reasons discussed in my course, this arrangement ensures pitch stability, i.e. the angle of attack tries to remain at the trimmed value. Increase the angle of attack, e.g. by pulling the nose up and then releasing the controls, and the nose will naturally want to come back down.

What happens if we turn the airplane upside down without changing anything?

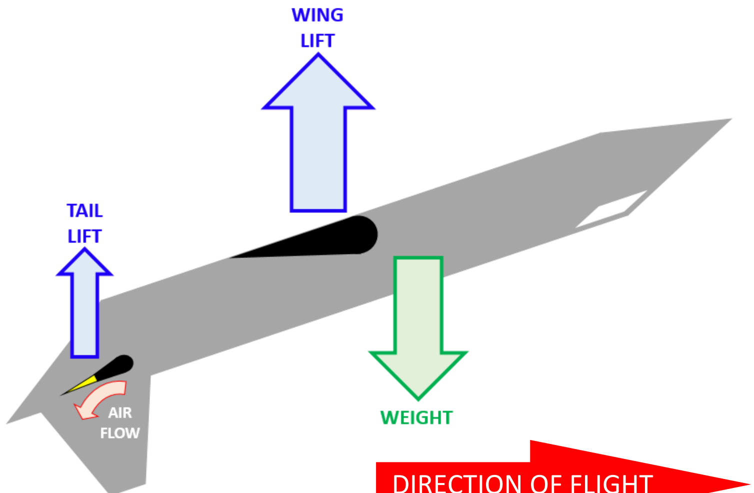
Both the weight and the wing lift will pull the airplane down! It will fall like a rock… or, actually, faster than a rock!

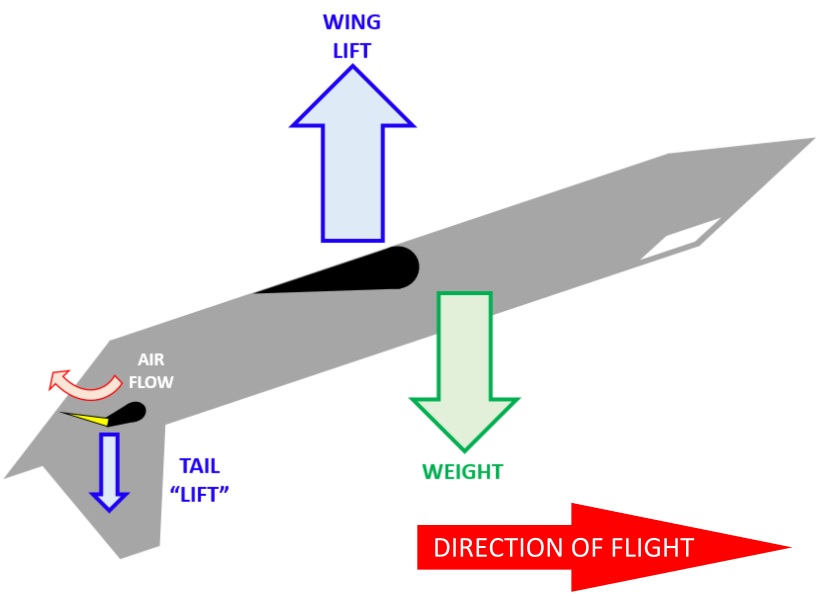


But what if we bring the nose up so that the wing is flying at a positive angle of attack, generating about as much lift as it was before? That will at least keep the airplane from falling right away. (In the image below, the airplane is flying to the right, not upwards. The fuselage is at a high angle of attack, but the wing is at the same moderate angle of attack as the first of these drawings, in the previous page).

We're almost there. The only problem is that the center of gravity is ahead of the center of lift, so this airplane will see-saw about the wings: The tail will rise and the nose will drop, putting the airplane into an inverted dive.

To solve this problem, we need down-force from the tail, just like we had during level flight (in the previous page). But the tail is set at an angle relative to the wings, pointing "down" (towards the airplane's belly, i.e. in the absolute "up" direction when the airplane is inverted).

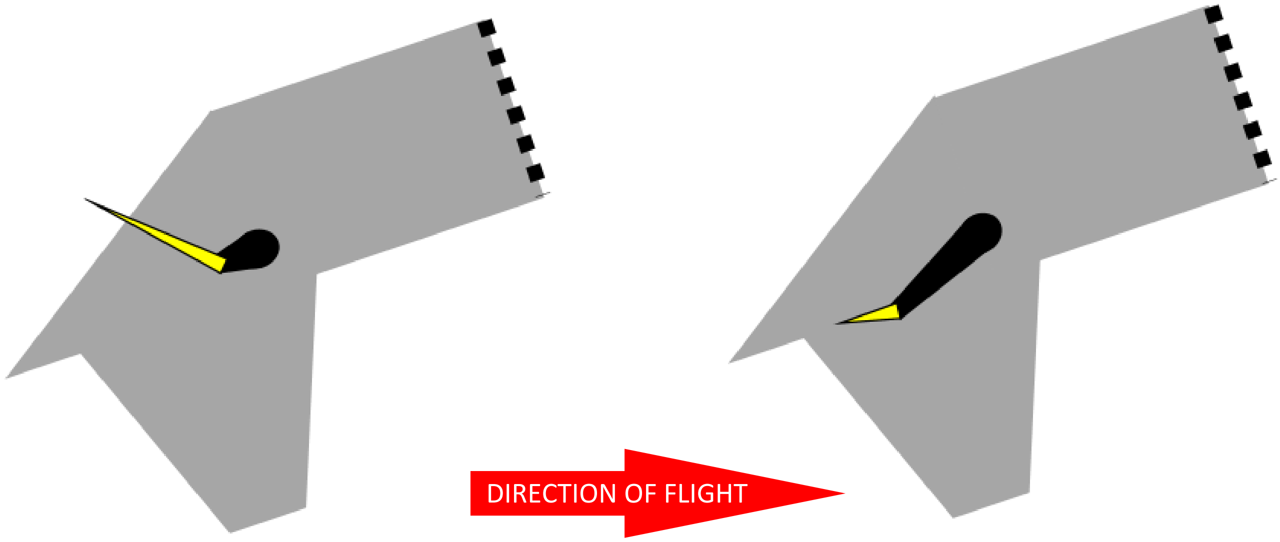




The solution: Deflect the elevators upwards as much as possible. Can the elevators be deflected up enough so that the combined "horizontal stabilizer plus elevator" airfoil is at an overall negative angle of attack? In other words, is the tail movable enough so that we can transition it from "deflecting air downwards" to "deflecting air upwards"?

In this final drawing, the elevators can be deflected up enough so that the combined "horizontal stabilizer plus elevator" airfoil is at an overall negative angle of attack. Therefore, the airplane is aerodynamically capable of sustaining -1g inverted flight.

However, as you can see below, this would not be the case if the elevators are small and/or if the horizontal stabilizer is set at a very negative angle relative to the fuselage during normal flight.



Probably capable of sustained inverted -1g flight

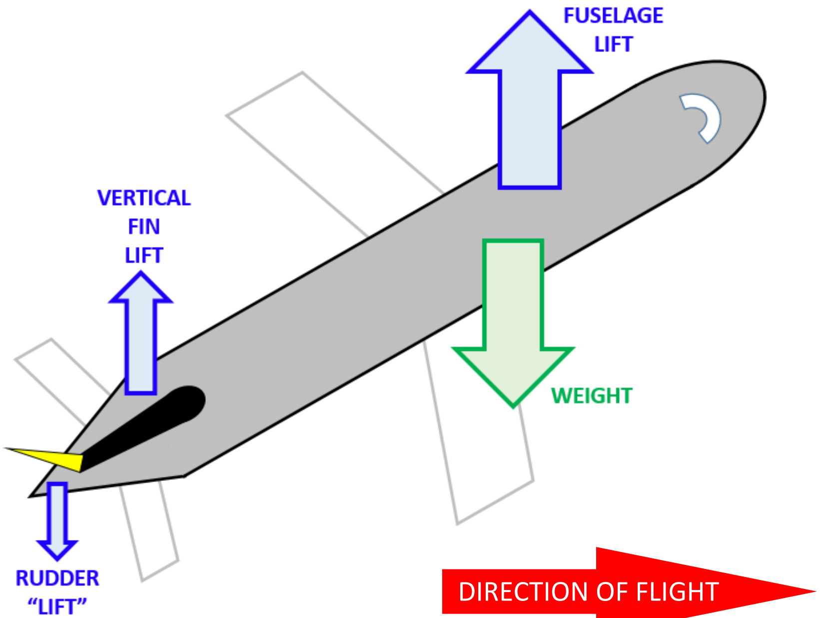
Probably not capable of sustained inverted -1g flight

In theory, both of these characteristics (small elevator, negatively angled horizontal fins) seem appropriate for slow, stable airplanes such as single-engine trainers and jet airliners. In practice, however, even jetliners and Cessnas do have enough elevator authority for -1g flight. I will leave it as an exercise to the reader to set up the math. What fraction of the horizontal stabilizer must be made up of a movable elevator, and what deflection angle is necessary for that elevator, to sustain inverted flight in an airplane where the CG is a certain distance ahead of the wings center of lift?

Another way to think about it: Imagine flying an airplane in normal horizontal flight, then pushing the yoke or joystick forward very hard, pushing the airplane into a dive, causing all loose objects (and occupants) inside to "fall up" to the ceiling. Some level of negative Gs were just pulled, less than zero. If the controls are pushed forwards all the way, how negative can those Gs get, in that airplane? It will help if the tail is trimmed all the way nose-down (which in jetliners means that the horizontal stabilizer itself is rotated upwards, or at least less-downwards) and if the speed is higher.

In the end, a given elevator deflection (including pitch trim of the elevators or of the horizontal stabilizers) gives a certain angle of attack. At slower speeds, you want a higher angle of attack in order to sustain 1g flight, so you either pull up on the controls all the time, or you use trim to deflect the entire horizontal stabilizer downwards (or to set the zero position of the elevator at a more upwards deflection angle). At faster speeds, you want a lower angle of attack in order to sustain 1g flight, so you either push down on the controls all the time, or you use trim to deflect the entire horizontal stabilizer upwards or less downwards (or to set the zero position of the elevator at a more downwards deflection angle). So pushing the controls all the way nose-down simply commands the minimum (most negative) angle of attack that the airplane can sustain. The faster you're going, the more negative Gs this angle of attack will supply. Go fast enough, and if you push down, you get -1g and you fly horizontally while inverted. Again, airliners and trainers are actually capable of this. One way to show this is to fly at about 70% of the max thrust speed (so that the airplane experiences about 50% of the dynamic pressure that it would see at max speed) and perform a -0.5g push-over. If the airplane can do -0.5g without stalling at 70% max speed, then it could sustain -1g flight at max speed.

This, of course, neglects the fact that the airplane is draggier while inverted. That is primarily because the wing is optimized for right-side-up , so it will generate more drag while inverted if generating the same lift force. It is also because the elevator will be deflected up like an air brake… unless the entire horizontal stabilizer is deflected down very much, either by a large amount of trim capability (which is rare because airplanes never need to sustain such low angles of attack "hands off") or by an all-moving stabilator (which is common in jet fighters and other supersonic airplanes but is relatively rare among slower airplanes, with exceptions like the Beech Musketeer and the RV-12).



Similar reasoning can be used to address the question of whether an airplane can sustain knife-edge flight, i.e. "flying on its side" so that the ***fuselage*** generates lift, rather than the wings.

The airplane will be at some angle of sideslip (i.e. sideways angle of attack). This means that the vertical stabilizer (a.k.a. the "vertical fin"), which is normally aligned with the airflow and generating no forces, will generate upwards lift and try to lift the tail higher into the air and cause the nose to drop below the horizon. This can be prevented if the rudder is relatively large and can be deflected by a large angle. How large? That depends on the size of the non-moving part of the vertical stabilizer, and on how far the center of gravity is forwards of the center of lift of the fuselage. The center of lift of the fuselage may be ahead of the center of gravity (if, like a wing, it is about ¼ of the way back from the front end) or it may be behind the center of gravity (since, as a long bluff body, it deflects air roughly similarly at all points along its length, so the center of lift might be about halfway down the fuselage, behind the center of gravity). If the center of lift is ahead of the center of gravity, then the vertical fin's lift will actually help balance the airplane, unless the tail makes too much or too little lift. The question, then – based on the shape of the fuselage – is whether the rudder is big enough, and has enough of an angle of travel, to generate whatever upwards or downwards lift is necessary to balance the airplane, i.e. to line up the total lift (from the fuselage, vertical fin, and rudder) with the center of gravity. The answer, in nearly all airplanes, is "No".

I will spare you the details, but while most airplanes (including airliners) are barely capable of sustained inverted -1g flight, they are not capable of knife-edge flight. Very high speeds are required to get sufficient lift out of the fuselage for keeping the airplane in the air, and to get sufficient force out of the rudder for it to hold the desired angle rather than being overpowered by the unbalanced lift distribution around the fuselage and vertical fin. Additionally, the enormous induced drag during knife-edge flight (because the fuselage is a very, very inefficient wing, with a very low aspect ratio and poor airfoil curvature) makes it impossible to sustain such high speeds except in extremely overpowered airplanes such as fighter jets and airshow aerobatic airplanes. Other airplanes would have to dive to well below their maximum safe speeds in order to have enough airflow to fly knife-edge. Before too many seconds, that dangerous extra speed would bleed off and the nose would start dropping. Again, modeling the specifics and getting some numbers is left as an exercise to the reader.