Predicting the Aerobatic Capabilities of Marginally Aerobatic Airplanes

Bernardo Malfitano – UnderstandingAirplanes.com
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Bernardo Malfitano

Academic
• B.S. Mechanical Engr., Stanford University
• M.S. Mechanical Engr., Columbia University
  Elective courses, lab work, and research topics included airplane design, aerodynamics, control systems, and propulsion

Hobbies
• Articles and photos on various aviation magazines, websites, & books, since 2003
• Pilot, RV-6 owner. 1st solo: 2009
  1st aerobatic solo: 2012
  1st flight to Oshkosh: 2014

Professional
• Boeing Commercial Aviation Services
  (Fleet support, structural analysis of repairs, maintenance planning); Long Beach: 2007-2008
• BCA Structural Damage Technology
  (Fatigue & Fracture Mechanics allowables testing and analysis methods development); Everett: 2009-2018
• BCA Airplane Configuration & Integration
  (Product Development); Harbour Pointe: 2018-Present
Notes

• All images are © their respective owners.

• All opinions expressed here are my own. I do not represent my employer, the FAA, or the EAA.

• A video of me delivering this talk can be found at https://www.youtube.com/channel/UCH7C3C5hKAVZR0SCPhECrMQ

• A paper that describes these physics principles in more detail is at https://www.understandingairplanes.com/Aerobatics-Analysis.pdf
Disclaimers

- I am not an FAA-certified instructor, or a test pilot!
- This is purposefully kept at a high-school level: For students, non-engineers, and EAA members ;]
- This talk is about what is physically possible, not what is safe, legal, or advisable.
DON'T DO ANYTHING STUPID!
Agenda

FAQ in my class:
Given an airplane...
• Can it do a roll?
• Can it do a loop?
What would be required?
Speed? Roll rate?
Ability to pull Gs?
**Spoiler Alert:**

- **Max Speed**
  - 200 kt
  - 100 kt
  - 0 kt

- **Roll OK**
  - 5 sec
  - 10 sec

- **Max Speed / Stall Speed**
  - Loop OK
    - Max G
      - 5
      - 4
      - 3
      - 2
      - 1

- **Time to roll 360°**
  - 1
  - 2
  - 3
  - 4
  - 5
  - 6
Rolls

- Slow Roll (-1g)
- Aileron Roll (~0g)
- Barrel Roll (>0g)
How Much Time at Zero G?
How Much Time at Zero G?

Enough to roll 360°?

Vertical component of velocity = Velocity \times \sin(\text{angle of climb})

Time at Zero G = \left( \frac{\text{Vertical component of velocity}}{g} \right) \times 2

Pull up to 30° (i.e. sine is half), **Zero G time is ~5 sec. per 100 kt**
So... Can I roll this airplane?

• Roll rate = ?
  • Find out by banking 90°R to 90°L (i.e. roll 180°).
  • Rolling 360° takes twice that long
  • Zero-g time must be longer than 360° roll time !!!

• Entry speed? Fast! (as long as no risk of hitting VNE).
  
  Go faster → More time at zero g & Faster roll rate.

• “Sharpening the pencil”: Some speed is lost during the pull-up due to induced drag from Gs and from altitude gain...
  but small impact. (Instead of 5 seconds per 100 kt, it’s really ~4.5sec).

• Bottom line: Roll is safe if zero-g time (~5 sec per 100kt) is higher than 360°-roll time (found empirically)
In Summary: Aileron Rolls

- Roll possible
- Roll not possible

Max Speed

- 200 kt
- 100 kt

Time to roll 360°

- 5 sec
- 10 sec
Aileron Rolls in Practice

• Most airplanes are physically capable of aileron rolls.
• Rebecca Wallick says most Boeing test pilots have rolled their prototype airliners.
Loops

• Very **tall** maneuver! Most K.E. at bottom becomes P.E. at top.

• Less so if you pull tighter... More Gs, more speed at top.

• **Key parameter = Ability to fly at several times the stall speed**

• G tolerance and engine thrust help, but not that much.
Kinetic and Potential Energy

- Say speed at bottom is $V_{\text{bottom}}$
- Loop height is $H$
- Speed at top is $V_{\text{top}}$
- How much lower than $V_{\text{top}}$ is $V_{\text{bottom}}$?

- Kinetic energy at the bottom is $\frac{1}{2} m V_{\text{bottom}}^2$
- Kinetic energy at the top is $\frac{1}{2} m V_{\text{top}}^2$
- Potential energy is $mgH$
- So $\frac{1}{2} m V_{\text{top}}^2 + mgH = \frac{1}{2} m V_{\text{bottom}}^2$
  
  i.e. $V_{\text{top}}^2 + 2gH = V_{\text{bottom}}^2$
Circular Motion

What is the relationship between

• Speed, \( V \)
• Centripetal acceleration (a.k.a. “Gs”), \( A \)
• and the local Radius of turn, \( R \)?

It’s \( A = \frac{V^2}{R} \)
3 Ways to Model a Loop

1: Circular Loop (constant radius)

• Very simple math and physics

• You need to be able to pull 6g!!!

• Not useful for “Can this airplane barely…”

\[ \frac{1}{2} V_{\text{bottom}}^2 = \frac{1}{2} V_{\text{top}}^2 + gH = \frac{1}{2} V_{\text{top}}^2 + g(2R) \]

\[ A_{Z,\text{top}} = 0 \]
\[ V_{\text{top}}^2 = 1gR \]
\[ V_{\text{bottom}}^2 = 1gR + 4gR \]
\[ \frac{V_{\text{bottom}}^2}{R} = 5g \text{ (plus gravity)} \]
3 Ways to Model a Loop

2: Constant or Increasing G Loop

(Cornu spirals and clothoids)

- Very hard math and physics
- Pretty accurate, but ignores loss of G capability below VA

\[ R = \frac{V_1^2 - 2gh}{A - G \cos \theta} \]

\[
\begin{align*}
\theta'(t) &= \kappa(t) \\
x(t) &= x_0 + \int_0^t \cos \left[ \theta_0 + \int_0^\tau \kappa(\sigma) \, d\sigma \right] \, d\tau \\
x'(t) &= \cos(\theta(t)) \\
y'(t) &= \sin(\theta(t)) \\
y(t) &= y_0 + \int_0^t \sin \left[ \theta_0 + \int_0^\tau \kappa(\sigma) \, d\sigma \right] \, d\tau
\end{align*}
\]
3 Ways to Model a Loop

3: Discretized Loop

(numerical analysis, circular segments)

- Very simple math and physics
- Arbitrarily accurate...
- ... if slightly less elegant.
What is Numerical Analysis

A “closed-form” or “exact solution” is:

• The most accurate way to solve a physics problem.
• You develop an equation and solve it exactly.
• But: Sometimes this is not possible.

A “numerical analysis” is:

• An approximate solution
• Not as accurate, but can get arbitrarily close
• You can get closer to the exact solution by...
  (1) brute computing power, and/or
  (2) tricks e.g. higher order methods
• Uses much simpler math, e.g. linear equations.
• Can be thought of as a “simulation”.

\[ \sigma_{xy}(y) = \frac{V}{2I} \left[ \left( \frac{h}{2} \right)^2 - y^2 \right] \]
Numerical Analysis of Loop

(assuming thrust=drag)
## Numerical Analysis of Loop

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### Input parameters
- Limit Load (G): 3
- Stall speed (mph): 65
- Speed at bottom (e.g. VNE): 160
- Maneuver speed: 112.583
- Increment size (degrees): 2
# Numerical Analysis of Loop

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<th>Segment</th>
<th>Start angle (degrees)</th>
<th>X start (m)</th>
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**Input parameters**

- Limit Load (G): 3
- Stalls speed (mph): 65
- Speed at bottom (e.g. VNE): 160
- Maneuver speed: 112.583
- Increment size (degrees): 2
The Result:

Blue region:
Possible without ever dropping below VA
(i.e. always max structural Gs)
In Practice...

MOST AIRPLANES

Max Speed / Stall Speed (e.g. VNE/VS1)

<table>
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<th>VS1</th>
<th>VNE</th>
<th>Ratio</th>
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Or rather: How “perfectly” must it be executed?

So while the most important parameter to being able to perform a loop at all is $VS1/VNE$ ratio, the most important parameter to being able to do it safely (i.e. going nowhere near the max structural $G$) is structural $G$ capability!

This also allows for more circular loops.
In Summary: Loops

- The higher the ratio between VNE and VS1, the fewer Gs are required to do a loop.
- The ability to pull more Gs makes it *a little* easier to do a loop, i.e. gives a wider margin of safety, and allows for a more circular loop (e.g. for aerobatic competitions).
Immelmans, Split-Ss, Cuban 8s

- Made of parts of loops and rolls.
- So, similar requirements.

- However...
Immelmans, Split-Ss, Cuban 8s

• Immelmans and Split-Ss need enough speed at the top of the half-loop to sustain 1g flight (i.e. more than VS1)

• So for an Immelman or a Split-S, more Gs – or more speed – is needed at the start, when compared with just a loop.

• Here, orange “barely” line is not “Speed barely above zero at 90° nose-up”, it’s “Speed above VS1 at very top of loop”.

• Basically the same as previous graph, but the line is moved “one to the right”, i.e. the airplane MUST enter the maneuver at over twice its stall speed.
Super Tall Loop

- Density changes with altitude...
- ...so stall speed, and maneuver speed, change with altitude.
### Super Tall Loop

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<th>Gs</th>
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**Input parameters**
- Limit Load (G): 2.2
- Stall speed at sea level: 220
- Speed at bottom (e.g., VNE): 550
- Increment size (degrees): 0.5
Super Tall Loop

Say...
• Max speed at sea level
  = 550 mph (478 kt)
• VS1 = 220 mph
• Max load = 2.2g

It works!
Airliner only doing 118 mph at top, stall is 254, i.e. just enough for ~0.2g!
(Is 550 mph at sea level too much? Ok: Bare minimum: 415 mph at sea level, or 550 mph starting at ~17,000 feet).
(If 2.5g instead of 2.2, and 180mph stall instead of 220: could start at 370 mph at sea level. In reality, >2.5g probably ok if not at MTOW).
Slow Rolls

- Level flight,
- knife-edge one way,
- -1g flight,
- knife-edge the other way,
- Back to level flight:

What airplanes can sustain -1g flight?
How about knife-edge?
Sustained Inverted Flight (-1g)

... requires three things:

1: Structural strength.
2: Systems, e.g. inverted oil pump and fuel hose
3: Sufficient elevator authority. (This is easier at faster speeds)
“Elevator Authority”?

• The first two are obvious. As for the third...
• Recall that the horizontal stabilizer must push down for balance.
“Elevator Authority”? 

• So for sustained inverted flight, we need...

No way! Will fall!

Nope. Nose will come down.

That’s it! 😊
“Elevator Authority”?

- So the key questions are: **Is the elevator big enough**, and does it deflect through **enough of an angle**, to change the horizontal tail lift from “upwards” to “downwards” when the airplane is at a highly negative angle of attack?

- Note: The faster the speed, the less angle is needed. (Can the airplane do a -1g push-over? If so, then it can sustain -1g flight. And it can, at SOME speed).

![Diagram showing elevator angle and direction of flight](image)
Knife Edge Flight

• Just as -1g capability depends on elevator authority, knife-edge capability depends on **rudder authority**.

• Both the CG and the fuselage’s center of lift will be somewhere near the front. But the vertical stabilizer will make a lot of lift, wanting to bring the tail up and the nose down. Can the rudder overcome this? Not on most airplanes.

• (i.e. : In airplanes capable of knife-edge flight, a maximum slip – full rudder one way, then bank the other way until the airplane is not turning – will become knife-edge).
Just Scratching the Surface

• We did not discuss some relatively simple maneuvers such as spins, hammerheads, or snap rolls. How could they be modeled?

• Around a loop, Thrust ≠ Drag. By how much? Depends on speed, Gs, relationship between entry speed and max level-flight speed... e.g. an F-22 loop looks very different from a glider loop!

• What about high-torque gyroscopic maneuvers like lomcevaks?

• How are things different in formation aerobatics?

• How about in airplanes that are neutrally stable or unstable, like a Sukhoi-27 performing a Cobra maneuver?

• What possibilities are opened up by thrust-vectoring jets and airplanes with large control surfaces placed in the prop-wash?

• But hopefully tonight’s presentation was enough to help you think about the capabilities of a homebuilt to do basic aerobatics.
Questions?