


Predicting the Aerobatic Capabilities of Marginally Aerobatic Airplanes

Bernardo Malfitano – UnderstandingAirplanes.com

EAA AirVenture Oshkosh 2018



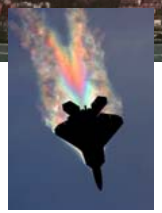

Bernardo Malfitano

Academic

- B.S. Mechanical Engr., Stanford University
- M.S. Mechanical Engr., Columbia University
  Elective courses, lab work, and research topics included airplane design, aerodynamics, control systems, and propulsion

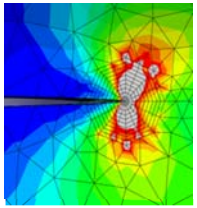
Hobbies

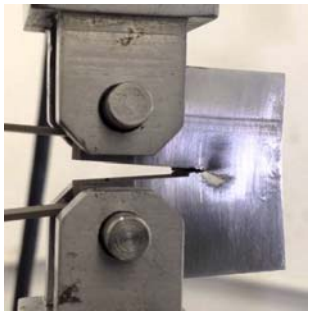
- Articles and photos on various aviation magazines, websites, & books, since 2003
- Pilot, RV-6 owner. 1st solo: 2009
  1st aerobatic solo: 2012
  1st flight to Oshkosh: 2014



Professional

- Boeing Commercial Aviation Services
  (Fleet support, structural analysis of repairs, maintenance planning); Long Beach: 2007-2008
- BCA Structural Damage Technology
  (Fatigue & Fracture Mechanics allowables testing and analysis methods development); Everett: 2009-2018
- BCA Airplane Configuration & Integration
  (Product Development); Harbour Pointe: 2018-Present



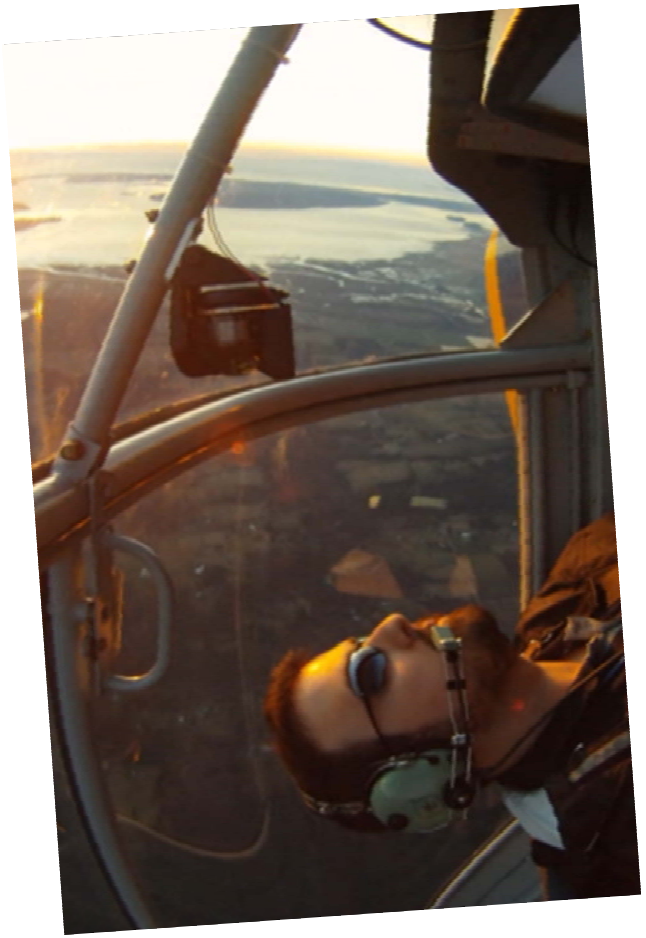


Notes

- All images are © their respective owners.
- All opinions expressed here are my own. I do not represent my employer, the FAA, or the EAA.
- A video of me delivering this talk can be found at https://www.youtube.com/channel/UCh7C3C5hKAVZR0SCPhECrMQ
- A paper that describes these physics principles in more detail is at https://www.understandingairplanes.com/Aerobatics-Analysis.pdf

Disclaimers

- I am not an FAA-certified instructor, or a test pilot!
- This is purposefully kept at a high-school level: For students, non-engineers, and EAA members ;]
- This talk is about what is ***physically possible***, not what is safe, legal, or advisable.



Agenda

FAQ in my class:

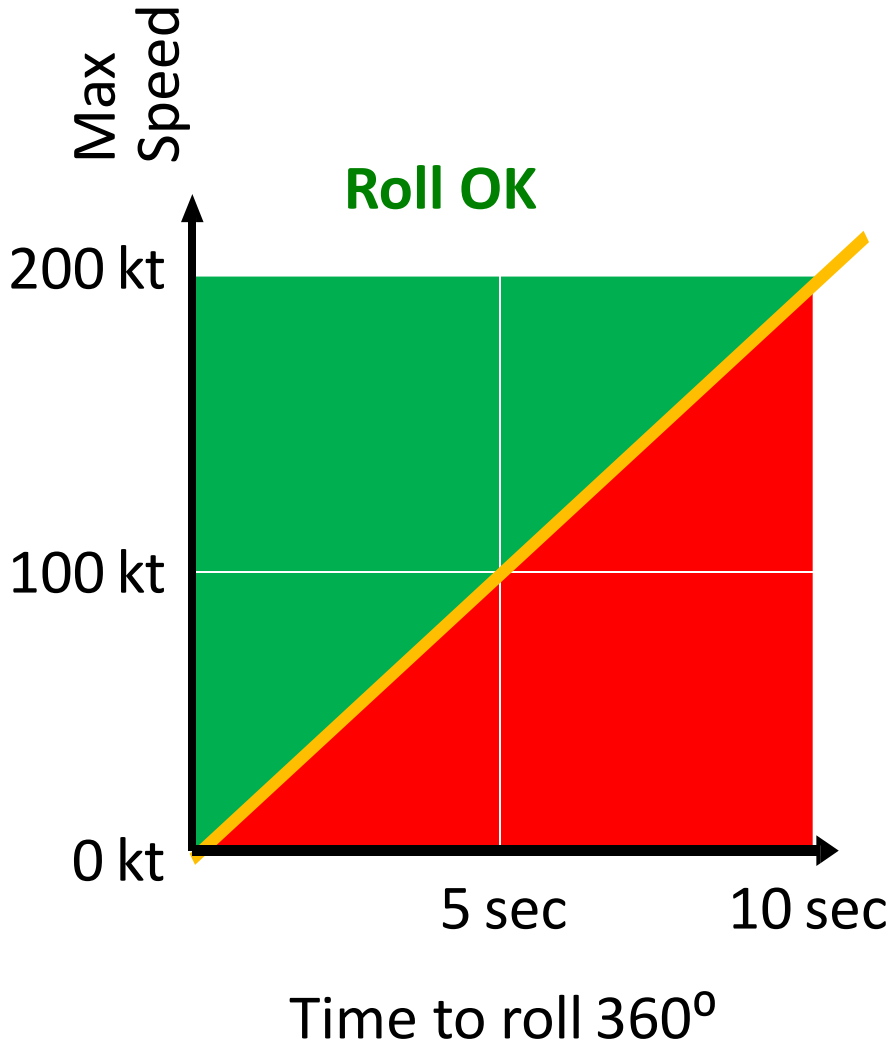
Given an airplane…

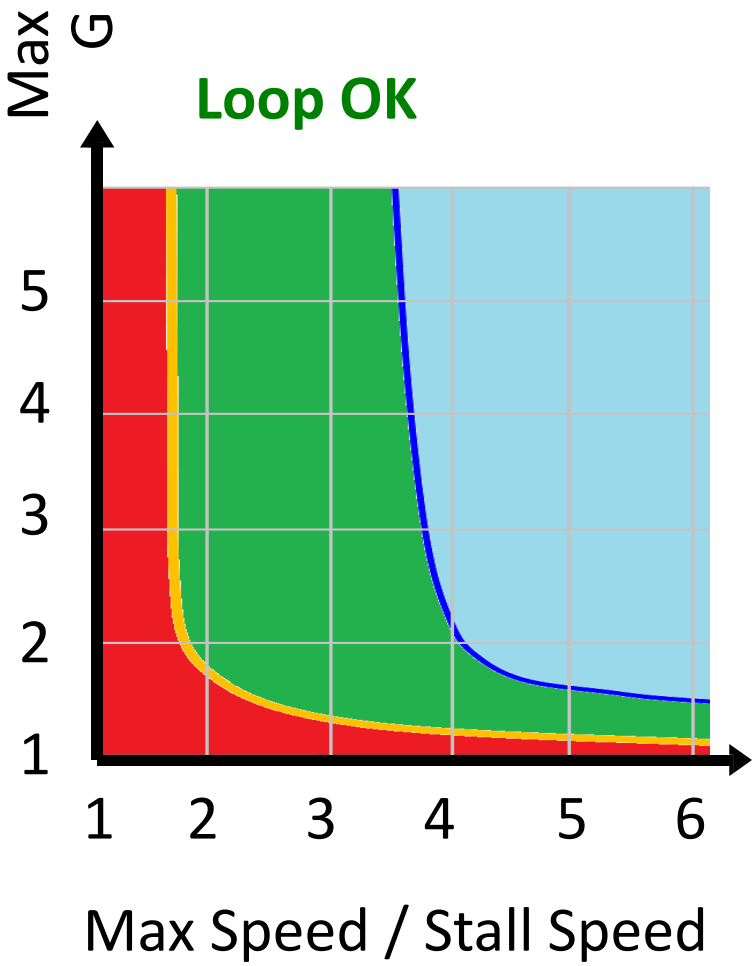
- Can it do a roll?
- Can it do a loop?

What would be required?

Speed? Roll rate?
Ability to pull Gs?

Spoiler Alert:

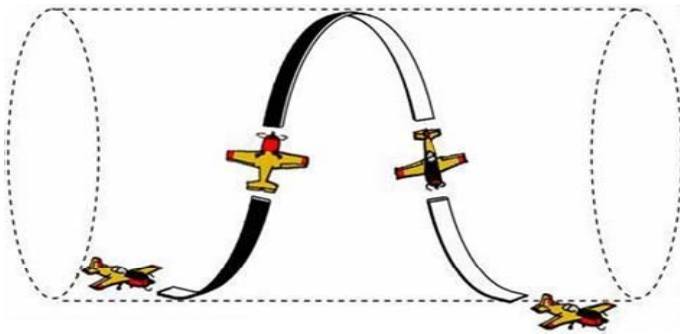




Rolls

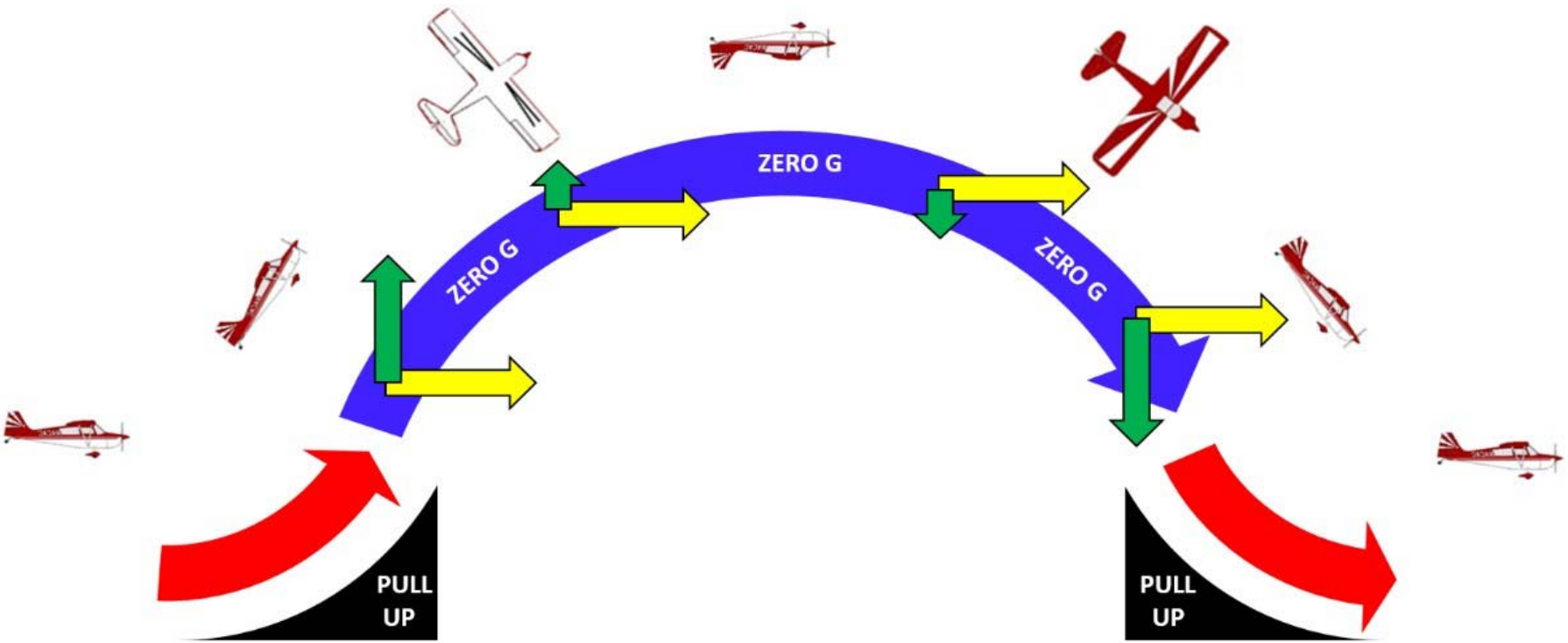
- Slow Roll (-1g)

- Aileron Roll (~0g)



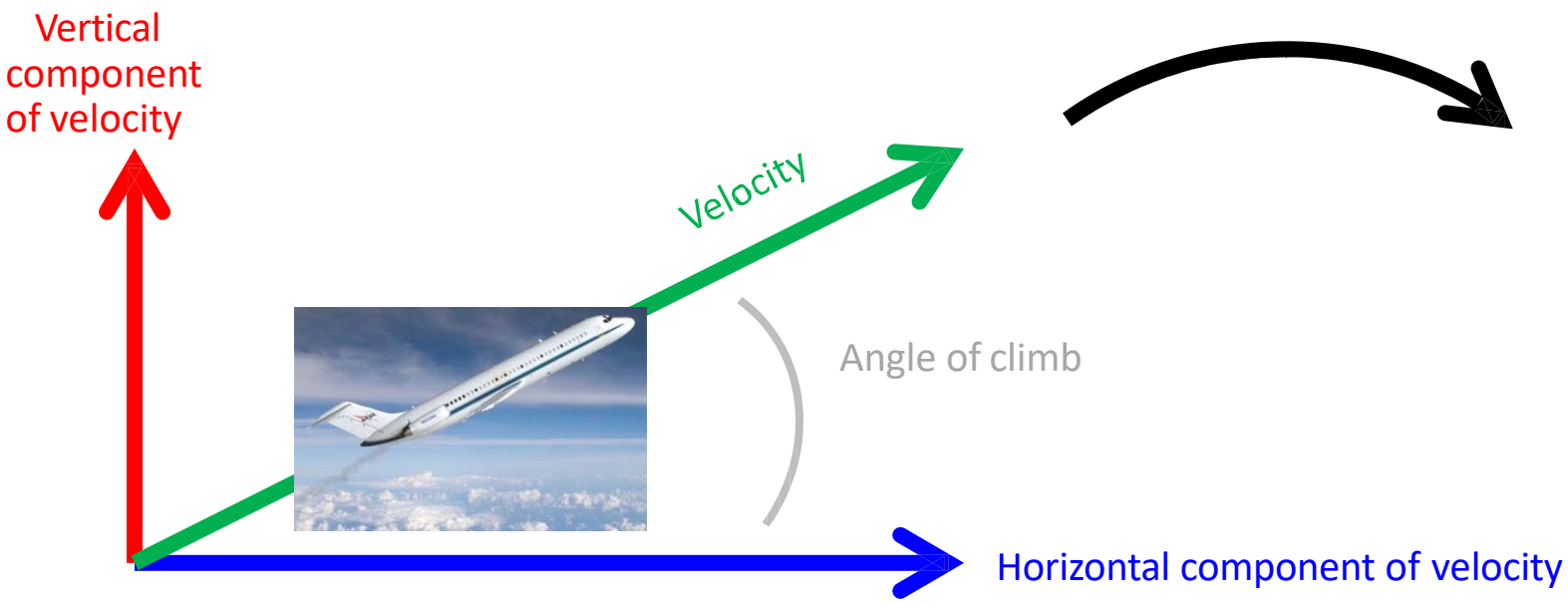
- Barrel Roll (>0g)

How Much Time at Zero G?



How Much Time at Zero G?

Enough to roll 360°?



Vertical component of velocity = Velocity x sine of angle of climb (vectors, trig)

Time at Zero G = (Vertical component of velocity / g) x 2 (definition of acceleration) (projectile/ballistic motion)

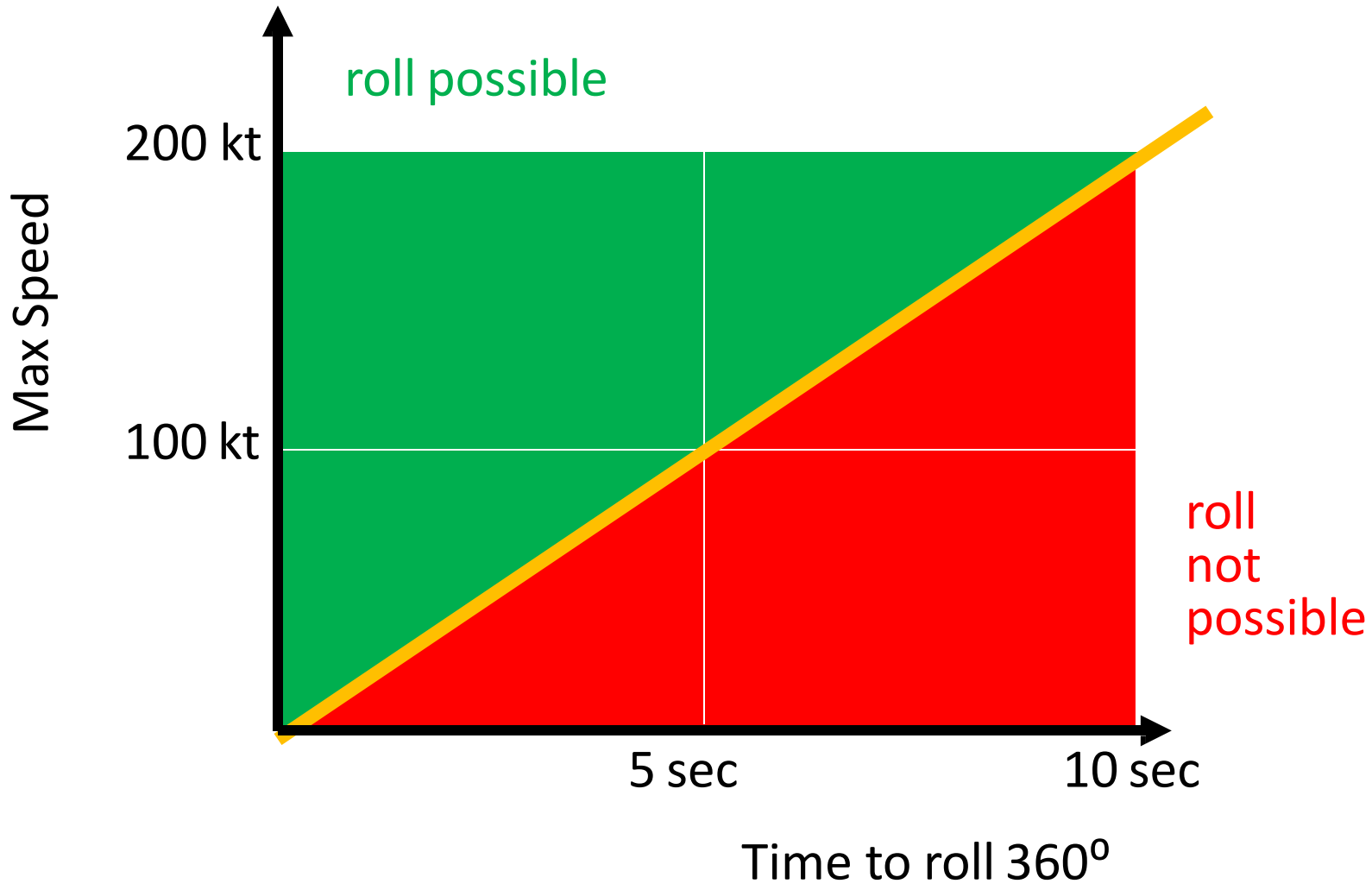
Pull up to 30° (i.e. sine is half), **Zero G time is ~5 sec. per 100 kt**

So… Can I roll this airplane?

- Roll rate = ?
  - Find out by banking 90°R to 90°L (i.e. roll 180°).
  - **Rolling 360° takes twice that long**
  - **Zero-g time must be longer than 360° roll time !!!**
- Entry speed? Fast! (as long as no risk of hitting VNE).
  Go faster → More time at zero g & Faster roll rate.
- "Sharpening the pencil": Some speed is lost during the pull-up due to induced drag from Gs and from altitude gain…
  but small impact. (Instead of 5 seconds per 100 kt, it's really ~4.5sec).

- Bottom line: ***Roll is safe if zero-g time*** (~5 sec per 100kt) ***is higher than 360°-roll time*** (found empirically)

In Summary: Aileron Rolls



Aileron Rolls in Practice

- Most airplanes are physically capable of aileron rolls.
- Rebecca Wallick says most Boeing test pilots have rolled their prototype airliners.

Loops

- Very *tall* maneuver!
  Most K.E. at bottom
  becomes P.E. at top.

- Less so if you pull tighter…
  More Gs, more speed at top.

- Key parameter = Ability to fly at several times the stall speed

- G tolerance and engine thrust help, but not that much.

Kinetic and Potential Energy

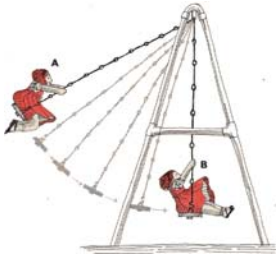
- Say speed at bottom is V_{bottom}
  loop height is H
  speed at top is V_{top}
- How much lower than V_{top} is V_{bottom}?

- Kinetic energy at the bottom is $\frac{1}{2} mV_{bottom}^2$

  Kinetic energy at the top is $\frac{1}{2} mV_{top}^2$
- Potential energy is mgH
- So $\frac{1}{2} mV_{top}^2 + mgH = \frac{1}{2} mV_{bottom}^2$

  i.e.

$$\mathbf{V_{top}^2 + 2gH = V_{bottom}^2}$$

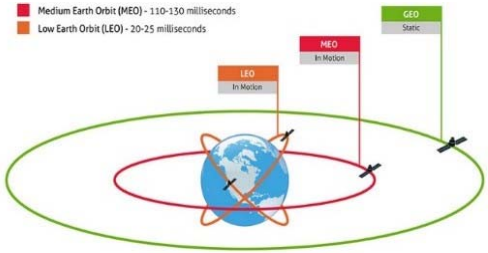


Circular Motion

What is the relationship between

- Speed, V
- Centripetal acceleration (a.k.a. “Gs”), A
- and the local Radius of turn, R?

It’s $\mathbf{A = V^2/R}$



3 Ways to Model a Loop

$$\tfrac{1}{2}V_{bottom}^2 = \tfrac{1}{2}V_{top}^2 + gH = \tfrac{1}{2}V_{top}^2 + g(2R)$$

1: Circular Loop (constant radius)

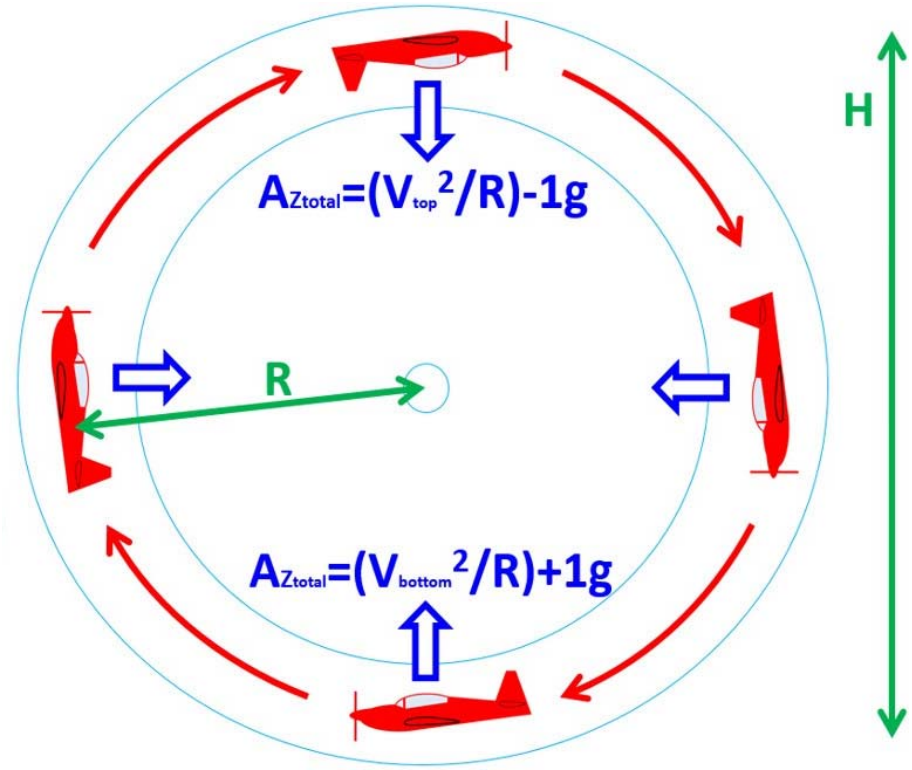
- Very simple math and physics
- You need to be able to pull 6g!!!
- Not useful for "Can this airplane barely…"

$A_{Ztop}=0$

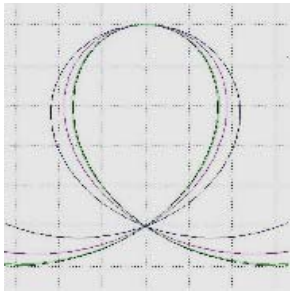
$V_{top}^2=1gR$

$V_{bottom}^2=1gR+4gR$

$V_{bottom}^2/R=5g$ (plus gravity)



3 Ways to Model a Loop

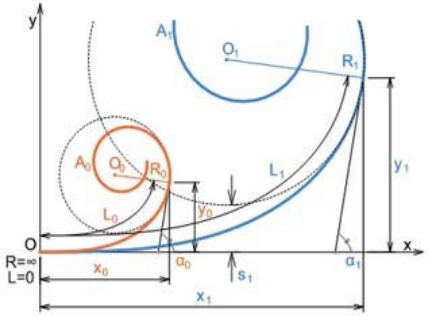


2: Constant or Increasing G Loop

(Cornu spirals and clothoids)

$$R = \frac{V_1^2 - 2gh}{A - G\cos\theta}$$

- Very hard math and physics
- Pretty accurate, but ignores loss of G capability below VA



$$\theta'(t) = \kappa(t)$$
$$x'(t) = \cos(\theta(t))$$
$$y'(t) = \sin(\theta(t))$$

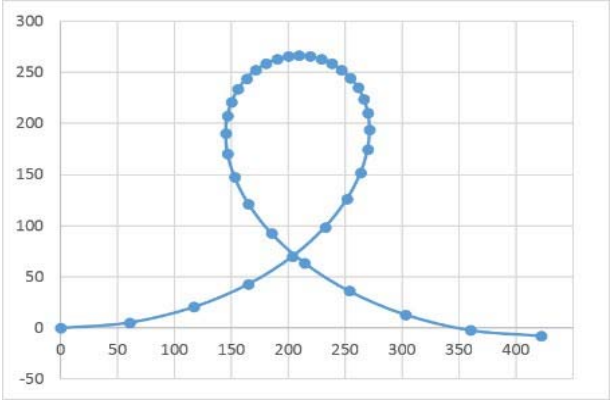
$$x(t) = x_0 + \int_0^t \cos\left[\theta_0 + \int_0^\tau \kappa(\sigma)\,d\sigma\right] d\tau$$
$$y(t) = y_0 + \int_0^t \sin\left[\theta_0 + \int_0^\tau \kappa(\sigma)\,d\sigma\right] d\tau$$

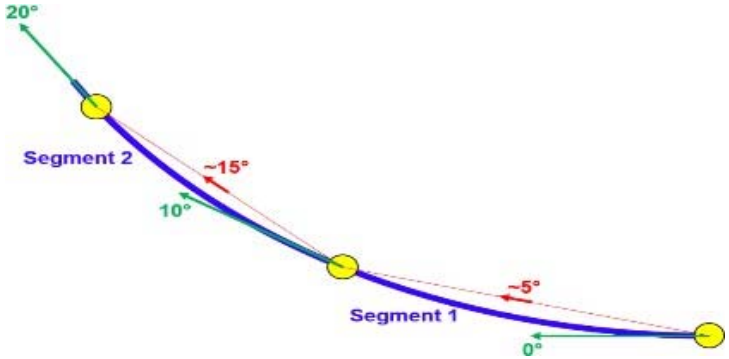
3 Ways to Model a Loop

3: Discretized Loop

(numerical analysis, circular segments)

- Very simple math and physics
- Arbitrarily accurate…
- … if slightly less elegant.





Segment	Start angle (degrees)	X at start (m)	H at start (m)	Start speed (mph)	Gs	Local Radius (m)	Segment Length (m)	Horizontal Component (m)	Vertical Component (m)	V at end (m/s)
0	0	0	0	160	3	260.2604674	45.36639	45.19375589	3.953941302	158.7822945
1	10	45.19376	3.95394	158.782295	3	250.5671339	43.67673	42.18848087	11.30436938	155.2481787
2	20	87.38224	15.2583	155.248179	3	229.2032901	39.95277	36.20950292	16.88476851	149.8142667
3	30	123.5917	32.1431	149.814267	3	202.1193249	35.23172	28.86013527	20.20808427	143.0397201
4	40	152.4519	52.3512	143.03972	3	173.2754477	30.2039	21.357383	21.357383	135.5122678
5	50	173.8093	73.7085	135.512268	3	145.7299381	25.4024	14.57021921	20.80842952	127.7524251
6	60	188.3795	94.517	127.752425	3	121.28158	21.14077	8.934476471	19.16004662	120.1649761
7	70	197.314	113.677	120.164976	3	100.6415847	17.54298	4.5404584	16.94522144	113.0309868
8	80	201.8544	130.622	113.030987	3	83.80097324	14.60747	1.273125083	14.55188629	106.5239646
9	90	203.1275	145.174	106.523965	2.6858	78.34991879	13.65729	-1.190311318	13.60532062	100.0581646
10	100	201.9372	158.779	100.058165	2.3696	72.66136583	12.66571	-3.278127127	12.23413699	93.86445107
11	110	198.6591	171.014	93.8644511	2.0853	66.92158004	11.6652	-4.929926559	10.57226162	88.1622973

What is Numerical Analysis

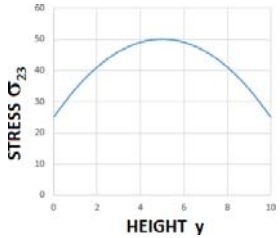
e.g. What is the shear stress in a beam, at each point along the height:

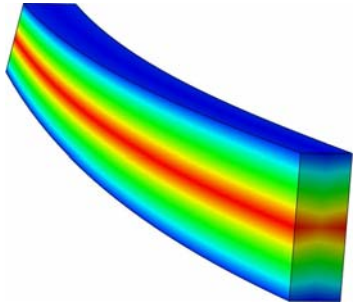
σ_{xy} = shear stress
y = height of each point
V = shear force
I = moment of inertia
h = height of beam

$$\sigma_{xy}(y) = \frac{V}{2I} \cdot \left[\left(\frac{h}{2}\right)^2 - y^2 \right]$$

A "closed-form" or "exact solution" is:

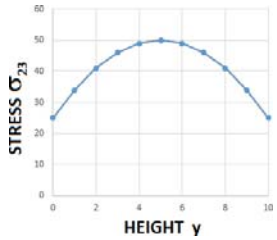
- The most accurate way to solve a physics problem.
- You develop an equation and solve it exactly.
- But: Sometimes this is not possible.

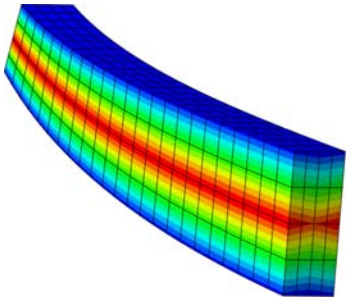




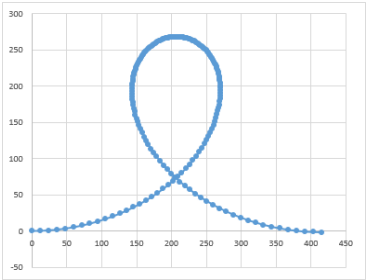
A "numerical analysis" is:

- An approximate solution
- Not as accurate, but can get arbitrarily close
- You can get closer to the exact solution by…
  (1) brute computing power, and/or
  (2) tricks e.g. higher order methods
- Uses *much* simpler math, e.g. linear equations.
- Can be thought of as a "***simulation***".





Numerical Analysis of Loop

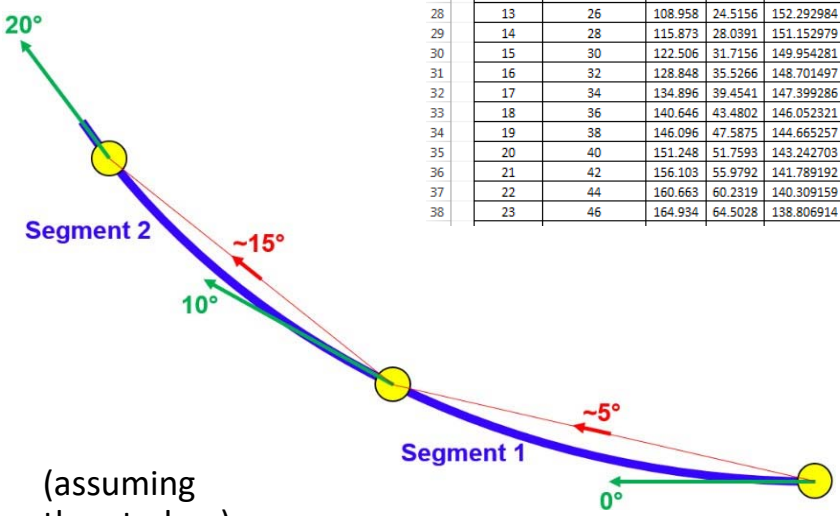


Input parameters	
Limit Load (G)	3
Stall speed (mph)	65
Speed at bottom (e.g. VNE)	160
Maneuver speed	112.583
Increment size (degrees)	2

Constants and Conversions	
m/s in an mph	0.44704
radians in a degree	0.017453293
one G in m/s²	9.81

0.434045916

Segment	Start angle (degrees)	X at start (m)	H at start (m)	Start speed (mph)	Gs	Local Radius (m)	Segment Length (m)	Horizontal Component (m)	Vertical Component (m)	1st iteration V at end (m/s)	2nd iteration Local Radius	2nd iteration Segment Length	2nd iteration Horizontal Component	2nd iteration Vertical Component
0	0	0	0	160	3	260.7357966	9.100934	9.099548073	0.158833202	159.9512624	260.65638	9.098162168	9.096776473	0.158784824
1	2	9.09678	0.15878	159.951277	3	260.3391003	9.087088	9.074634042	0.475581418	159.8052575	260.180738	9.081559976	9.069114014	0.475292126
2	4	18.1659	0.63408	159.805346	3	259.3905622	9.053979	9.019525912	0.789106268	159.5627691	259.233641	9.048501736	9.014069455	0.78862889
3	6	27.18	1.42271	159.562916	3	257.9776179	9.00466	8.937541105	1.09739206	159.2249552	257.823072	8.9992661	8.932186935	1.096734651
4	8	36.1121	2.51944	159.225158	3	256.1121332	8.939546	8.829485439	1.39845311	158.7934384	255.960859	8.934265891	8.824270252	1.397627105
5	10	44.9364	3.91707	158.793694	3	253.8095947	8.859176	8.696408385	1.69041055	158.2702723	253.662435	8.854039828	8.691366178	1.689430445
6	12	53.6278	5.6065	158.270576	3	251.0888342	8.764209	8.53958267	1.971518004	157.6579181	250.946567	8.759242972	8.534744142	1.970400942
7	14	62.1625	7.5769	157.658266	3	247.9717015	8.655406	8.360480038	2.240183875	156.9592179	247.83503	8.650635347	8.355872095	2.23894918
8	16	70.5184	9.81585	156.959604	3	244.482693	8.533623	8.160743926	2.494989802	156.1773628	244.352236	8.529069063	8.156389309	2.493658462
9	18	78.6748	12.3095	156.177781	3	240.6485481	8.399793	7.942159876	2.734704955	155.3158602	240.524834	8.395474308	7.93807691	2.733299076
10	20	86.6129	15.0428	155.316305	3	236.4978242	8.254912	7.706624544	2.958295994	154.3784965	236.381287	8.250844585	7.702827007	2.956838256
11	22	94.3157	17.9996	154.37896	3	232.060461	8.100027	7.456114136	3.164932678	153.3693001	231.951439	8.096221583	7.452611262	3.163445796
12	24	101.768	21.1631	153.369776	3	227.367345	7.936215	7.19265312	3.353989233	152.2925023	227.266082	7.932680052	7.189449703	3.352495454
13	26	108.958	24.5156	152.292984	3	222.4498839	7.764572	6.918283929	3.525041729	151.1524982	222.356529	7.76131302	6.915380537	3.523562377
14	28	115.873	28.0391	151.152979	3	217.3396003	7.586198	6.635038341	3.677861809	149.9538077	217.254213	7.583217668	6.632431616	3.676416878
15	30	122.506	31.7156	149.954281	3	212.0677514	7.402185	6.34491109	3.812407205	148.7010377	211.990311	7.399482121	6.342594116	3.811015027
16	32	128.848	35.5266	148.701497	3	206.6649828	7.213603	6.049836148	3.928809532	147.3988456	206.595389	7.211173379	6.047798873	3.92748651
17	34	134.896	39.4541	147.399286	3	201.161018	7.021488	5.751665995	4.027359887	146.0519041	201.099103	7.019326545	5.749895689	4.026120305
18	36	140.646	43.4802	146.052321	3	195.5843904	6.826837	5.452154089	4.108492796	144.6648684	195.529926	6.824935468	5.450635818	4.107348697
19	38	146.096	47.5875	144.665257	3	189.9622169	6.630596	5.15294062	4.172769038	143.2423466	189.914923	6.628944856	5.151657724	4.171730169
20	40	151.248	51.7593	143.242703	3	184.3200145	6.433656	4.855541529	4.220857856	141.7888714	184.279569	6.432243888	4.854476085	4.219931679
21	42	156.103	55.9792	141.789192	3	178.6815593	6.236846	4.561340696	4.253519012	140.3088758	178.647607	6.235661281	4.56047396	4.252710767
22	44	160.663	60.2319	140.309159	3	173.0687862	6.040934	4.271585112	4.271585112	138.8066712	173.040944	6.039961771	4.270897926	4.270897926
23	46	164.934	64.5028	138.806914	3	167.5017259	5.846616	3.987382796	4.275944546	137.2864284	167.479595	5.845843907	3.986855958	4.27537958



(assuming thrust=drag)

Cell Code.txt - Notepad

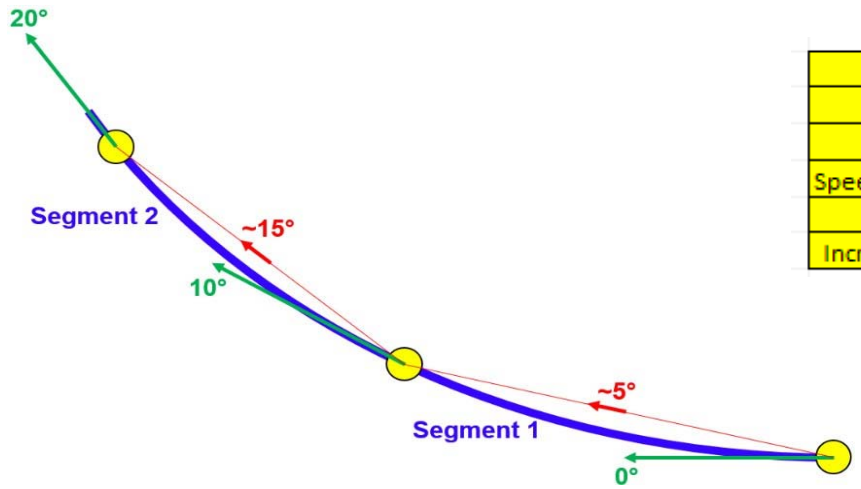
File Edit Format View Help

```
B16=B15+1
C16=B16*$D$12
D16=D15+O15
E16=E15+P15
F16=(1/$H$8)*SQRT((($H$8*$D$10)^2)-2*$H$10*E16)
G16=IF(F16>$D$11,$D$8,((F16)/$D$9)^2)
H16=(($H$8*(F16-0.5*(F15-F16)))^2)/($H$10*(G16-COS((C16+0.5*$D$12)*$H$9)))
I16=2*H16*SIN($D$12*$H$9/2)
J16=I16*COS(($C16+0.5*$D$12)*$H$9)
K16=I16*SIN(($C16+0.5*$D$12)*$H$9)
L16=(1/$H$8)*SQRT((($H$8*$D$10)^2)-2*$H$10*(E16+K16))
M16=(($H$8*AVERAGE(F16,L16))^2)/($H$10*(G16-COS((C16+0.5*$D$12)*$H$9)))
N16=2*M16*SIN($D$12*$H$9/2)
O16=N16*COS(($C16+0.5*$D$12)*$H$9)
P16=N16*SIN(($C16+0.5*$D$12)*$H$9)
```

Numerical Analysis of Loop

SEGMENT # | ANGLE (START) | X (START) | Y (START) | SPEED (START) | MAX Gs AT THIS SPEED | LOCAL RADIUS | ARC LENGTH | SEGMENT ΔX | SEGMENT ΔY | V AT END

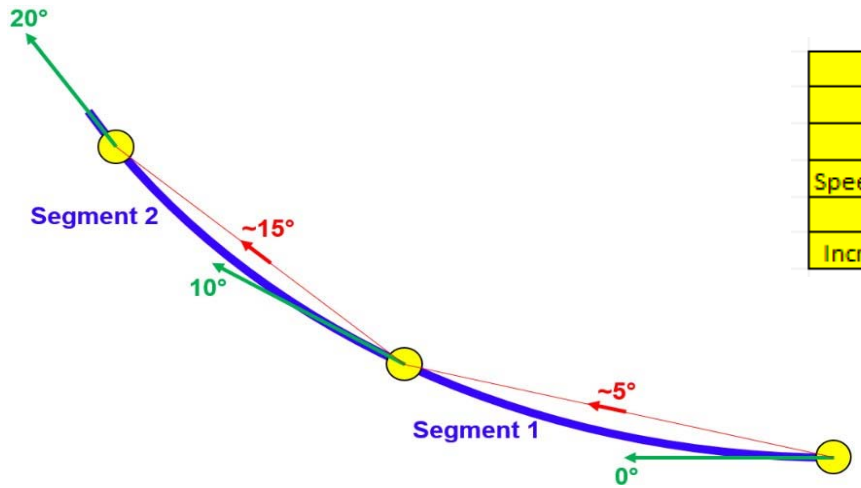
Segment	Start angle (degrees)	X at start (m)	H at start (m)	Start speed (mph)	Gs	Local Radius (m)	Segment Length (m)	Horizontal Component (m)	Vertical Component (m)	1st iteration V at end (m/s)
0	0	0	0	160	3	260.7357966	9.100934	9.099548073	0.158833202	159.9512624
1	2	9.09678	0.15878	159.951277	3	260.3391003	9.087088	9.074634042	0.475581418	159.8052575
2	4	18.1659	0.63408	159.805346	3	259.3905622	9.053979	9.019525912	0.789106268	159.5627691
3	6	27.18	1.42271	159.562916	3	257.9776179	9.00466	8.937541105	1.09739206	159.2249552
4	8	36.1121	2.51944	159.225158	3	256.1121332	8.939546	8.829485439	1.39845311	158.7934384
5	10	44.9364	3.91707	158.793694	3	253.8095947	8.859176	8.696408385	1.69041055	158.2702723
6	12	53.6278	5.6065	158.270576	3	251.0888342	8.764209	8.53958267	1.971518004	157.6579181
7	14	62.1625	7.5769	157.658266	3	247.9717015	8.655406	8.360480038	2.240183875	156.9592179
8	16	70.5184	9.81585	156.959604	3	244.482693	8.533623	8.160743926	2.494989802	156.1773628
9	18	78.6748	12.3095	156.177781	3	240.6485481	8.399793	7.942159876	2.734704955	155.3158602
10	20	86.6129	15.0428	155.316305	3	236.4978242	8.254912	7.706624544	2.958295994	154.3784965
11	22	94.3157	17.9996	154.37896	3	232.060461	8.100027	7.456114136	3.164932678	153.3693001
12	24	101.768	21.1631	153.369776	3	227.367345	7.936215	7.19265312	3.353089233	152.2925023



Input parameters	
Limit Load (G)	3
Stall speed (mph)	65
Speed at bottom (e.g. VNE)	160
Maneuver speed	112.583
Increment size (degrees)	2

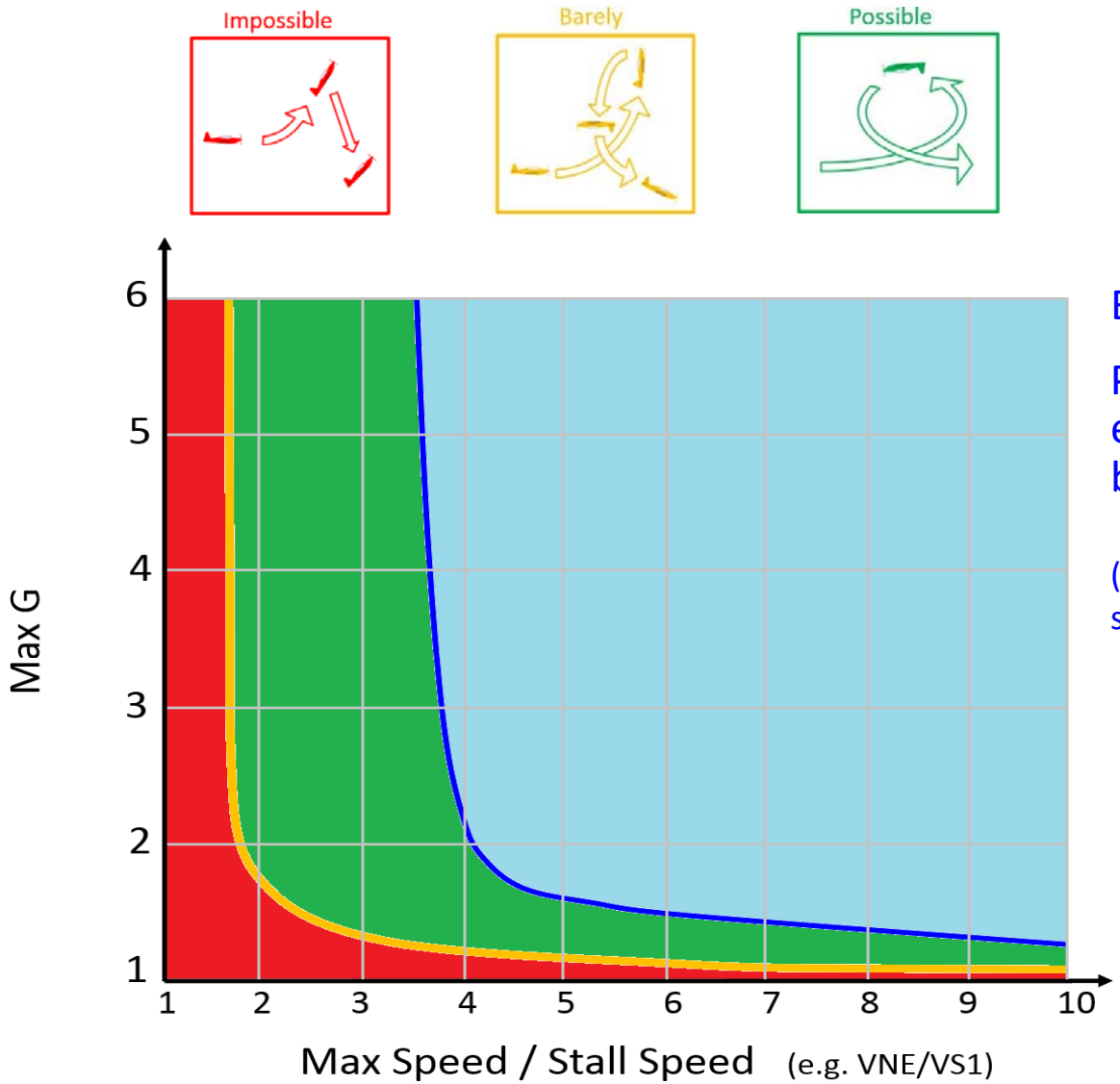
Numerical Analysis of Loop

SEGMENT #	ANGLE (START)	X (START)	Y (START)	SPEED (START)	MAX Gs AT THIS SPEED	LOCAL RADIUS	ARC LENGTH	SEGMENT ΔX	SEGMENT ΔY	V AT END	RADIUS W AVE. SPEED	NEW ARC LENGTH	NEW ΔX	NEW ΔY
Segment	Start angle (degrees)	X at start (m)	H at start (m)	Start speed (mph)	Gs	Local Radius (m)	Segment Length (m)	Horizontal Component (m)	Vertical Component (m)	1st iteration V at end (m/s)	2nd iteration Local Radius	2nd iteration Segment Length	2nd iteration Horizontal Component	2nd iteration Vertical Component
0	0	0	0	160	3	260.7357966	9.100934	9.099548073	0.158833202	159.9512624	260.65638	9.098162168	9.096776473	0.158784824
1	2	9.09678	0.15878	159.951277	3	260.3391003	9.087088	9.074634042	0.475581418	159.8052575	260.180738	9.081559976	9.069114014	0.475292126
2	4	18.1659	0.63408	159.805346	3	259.3905622	9.053979	9.019525912	0.789106268	159.5627691	259.233641	9.048501736	9.014069455	0.78862889
3	6	27.18	1.42271	159.562916	3	257.9776179	9.00466	8.937541105	1.09739206	159.2249552	257.823072	8.9992661	8.932186935	1.096734651
4	8	36.1121	2.51944	159.225158	3	256.1121332	8.939546	8.829485439	1.39845311	158.7934384	255.960859	8.934265891	8.824270252	1.397627105
5	10	44.9364	3.91707	158.793694	3	253.8095947	8.859176	8.696408385	1.69041055	158.2702723	253.662435	8.854039828	8.691366178	1.689430445
6	12	53.6278	5.6065	158.270576	3	251.0888342	8.764209	8.53958267	1.971518004	157.6579181	250.946567	8.759242972	8.534744142	1.970400942
7	14	62.1625	7.5769	157.658266	3	247.9717015	8.655406	8.360480038	2.240183875	156.9592179	247.83503	8.650635347	8.355872095	2.23894918
8	16	70.5184	9.81585	156.959604	3	244.482693	8.533623	8.160743926	2.494989802	156.1773628	244.352236	8.529069063	8.156389309	2.493658462
9	18	78.6748	12.3095	156.177781	3	240.6485481	8.399793	7.942159876	2.734704955	155.3158602	240.524834	8.395474308	7.93807691	2.733299076
10	20	86.6129	15.0428	155.316305	3	236.4978242	8.254912	7.706624544	2.958295994	154.3784965	236.381287	8.250844585	7.702827007	2.956838256
11	22	94.3157	17.9996	154.37896	3	232.060461	8.100027	7.456114136	3.164932678	153.3693001	231.951439	8.096221583	7.452611262	3.163445796
12	24	101.768	21.1631	153.369776	3	227.367345	7.936215	7.19265312	3.353989233	152.2925023	227.266082	7.932689052	7.189449703	3.352405454



Input parameters	
Limit Load (G)	3
Stall speed (mph)	65
Speed at bottom (e.g. VNE)	160
Maneuver speed	112.583
Increment size (degrees)	2

The Result:

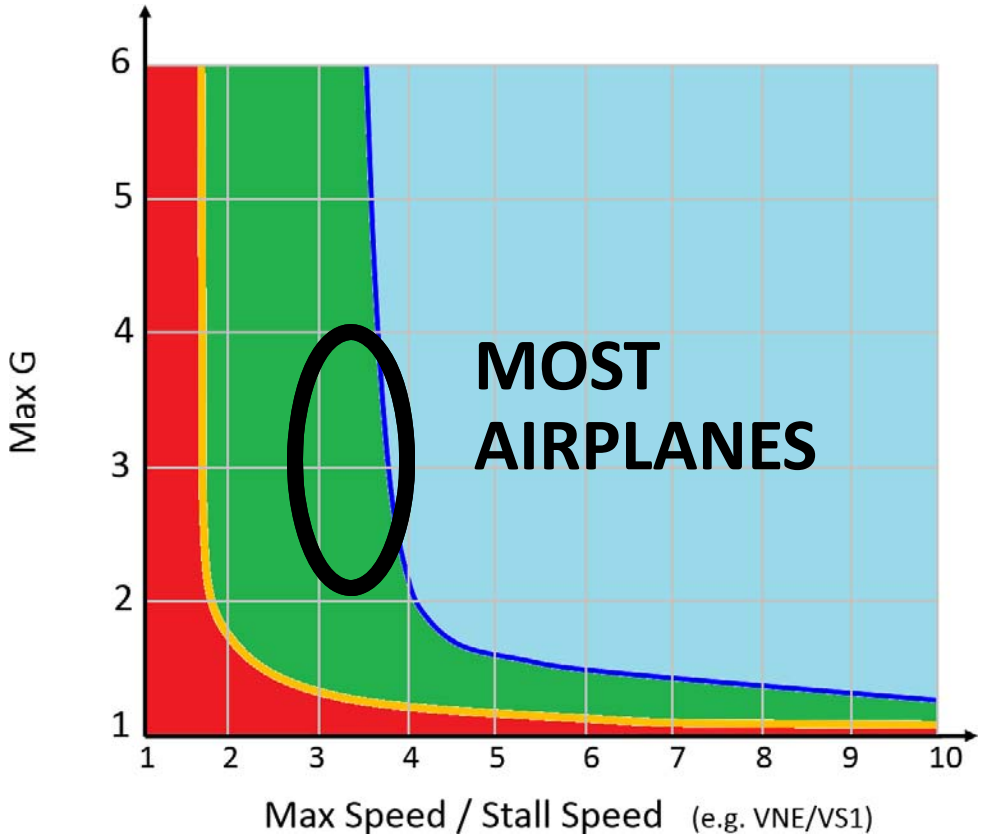


Blue region:

Possible without ever dropping below VA

(i.e. always max structural Gs)

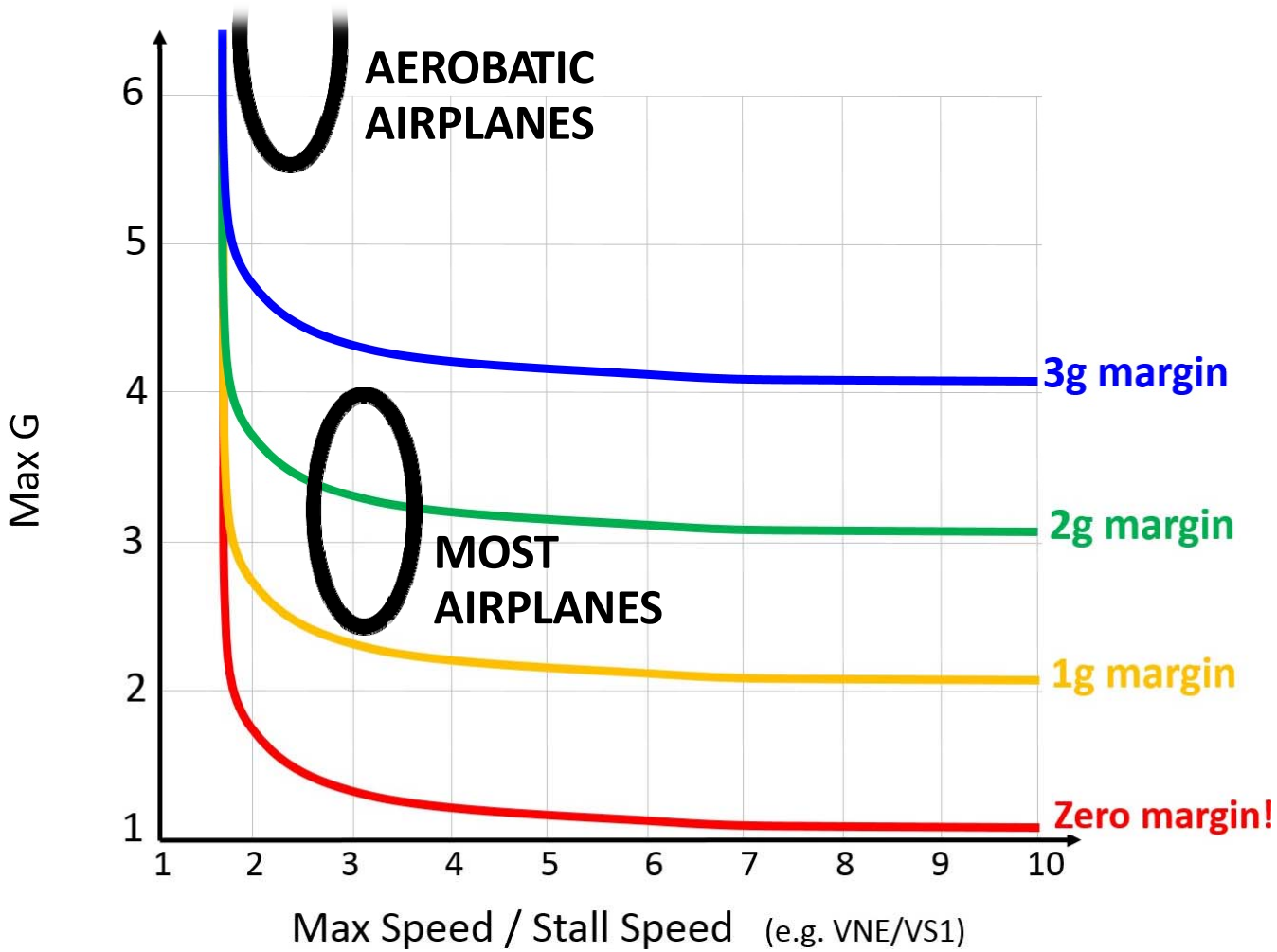
In Practice…



	Van's RV-6	Cirrus SR22	Beech A36 Bonanza	Piper J-3 Cub	Cessna 172	Lancair 4P	SportCruiser LSA	LearJet 23
VS1	65	70	68	38	57	69	32	104
VNE	185	201	205	121	182	274	138	561
Ratio	2.85	2.87	3.01	3.18	3.19	3.97	4.31	5.39

But: How safe is it?

Or rather: How "perfectly" must it be executed?



So while the most important parameter to being able to perform a loop at all is VS1/VNE ratio, the most important parameter to being able to do it ***safely*** (i.e. going nowhere near the max structural G) is structural G capability!

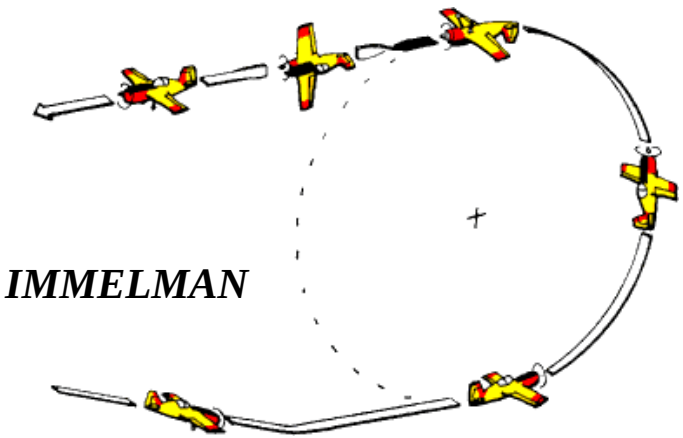
This also allows for more circular loops.

In Summary: Loops

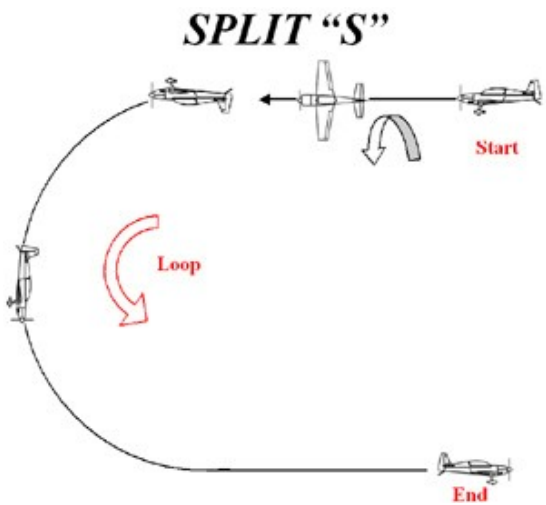
- The higher the ratio between VNE and VS1, the fewer Gs are required to do a loop.
- The ability to pull more Gs makes it ***a little*** easier to do a loop, i.e. gives a wider margin of safety, and allows for a more circular loop (e.g. for aerobatic competitions).

Immelmans, Split-Ss, Cuban 8s

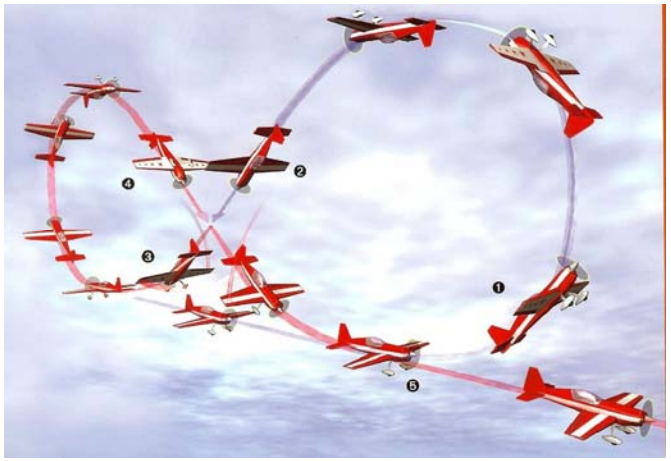
- Made of parts of loops and rolls.
- So, similar requirements.



IMMELMAN



- However…

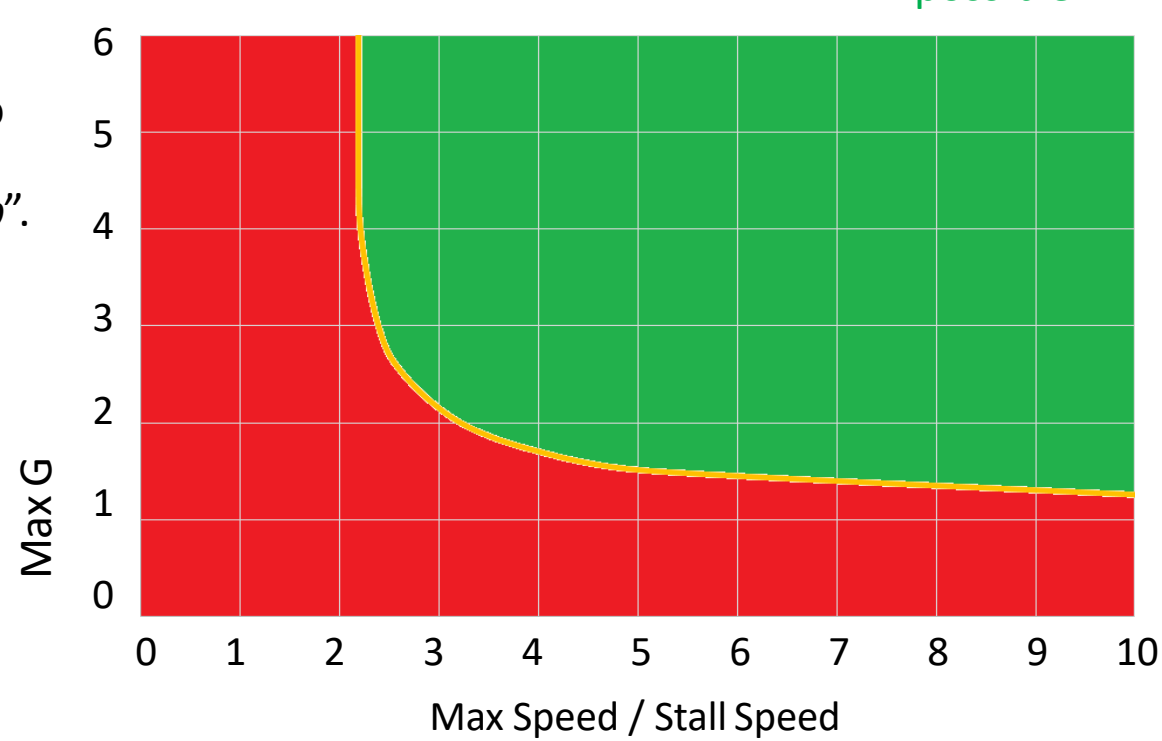


CUBAN 8

Immelmans, Split-Ss, Cuban 8s

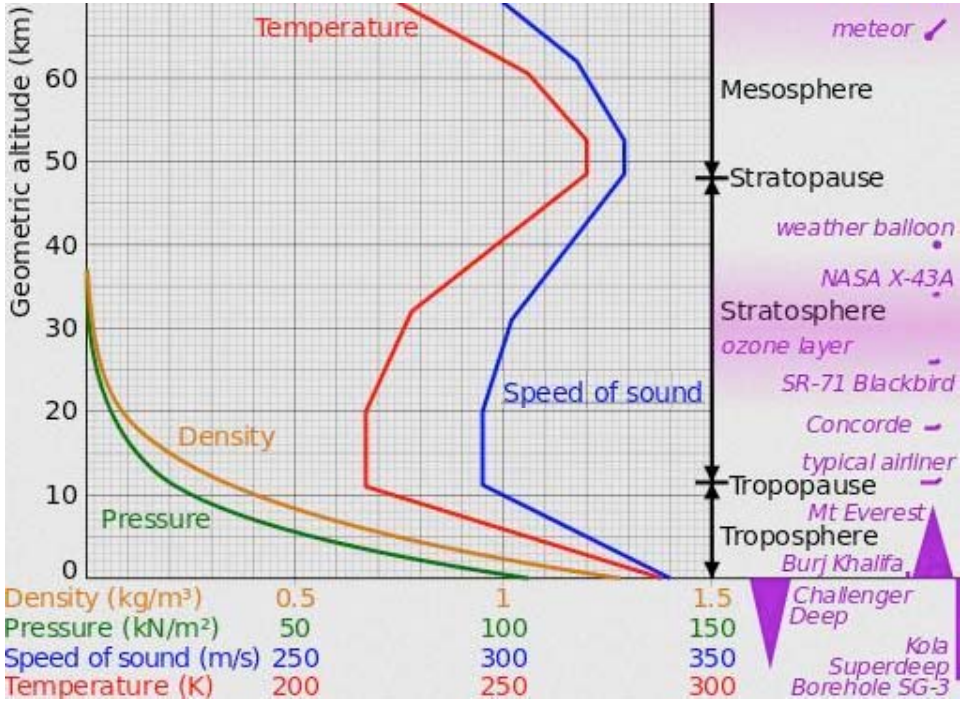
- Immelmans and Split-Ss need enough speed at the top of the half-loop to sustain 1g flight (i.e. more than VS1)
- So for an Immelman or a Split-S, more Gs – or more speed – is needed at the start, when compared with just a loop.

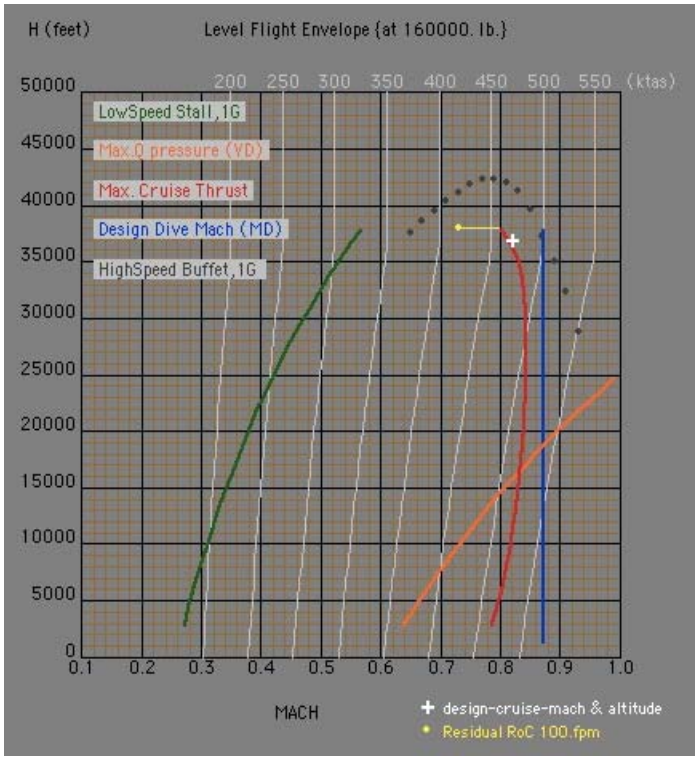
- Here, orange "barely" line is not "Speed barely above zero at 90° nose-up", it's "***Speed above VS1 at very top of loop***".
- Basically the same as previous graph, but the line is moved "one to the right", i.e. the ***airplane MUST enter the maneuver at over twice its stall speed.***



Super Tall Loop

- Density changes with altitude…
- … so stall speed, and maneuver speed, change with altitude.





(A320 sim)

Super Tall Loop

Untitled - Notepad

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```
B11=B10+1
C11=B11*$D$6
D11=D10+O10
E11=E10+P10
F11=(1/$H$3)*SQRT(((($H$3*$D$5)^2)-2*$H$5*E11)
G11=IF(F11>S11,$D$3,(F11/R11)^2)
H11=((($H$3*(F11-0.5*(F10-F11)))^2)/($H$5*(G11-COS((C11+0.5*$D$6)*$H$4)))
I11=2*H11*SIN($D$6*$H$4/2)
J11=I11*COS(($C11+0.5*$D$6)*$H$4)
K11=I11*SIN(($C11+0.5*$D$6)*$H$4)
L11=(1/$H$3)*SQRT(((($H$3*$D$5)^2)-2*$H$5*(E11+K11))
M11=((($H$3*AVERAGE(F11,L11))^2)/($H$5*(G11-COS((C11+0.5*$D$6)*$H$4)))
N11=2*M11*SIN($D$6*$H$4/2)
O11=N11*COS(($C11+0.5*$D$6)*$H$4)
P11=N11*SIN(($C11+0.5*$D$6)*$H$4)
Q11=(2116*(((59-0.00356*(E11+$H$7*$H$6)/$H$6)+459.7)/518.6)^5.256)/(1718*((59-0.00356*(E11+$H$7*$H$6)/$H$6)+459.7))
R11=$D$4/SQRT(Q11/0.00237692987919529)
S11=R11*SQRT($D$3)
```

Input parameters	
Limit Load (G)	2.2
Stall speed at sea level	220
Speed at bottom (e.g. VNE)	550
Increment size (degrees)	0.5

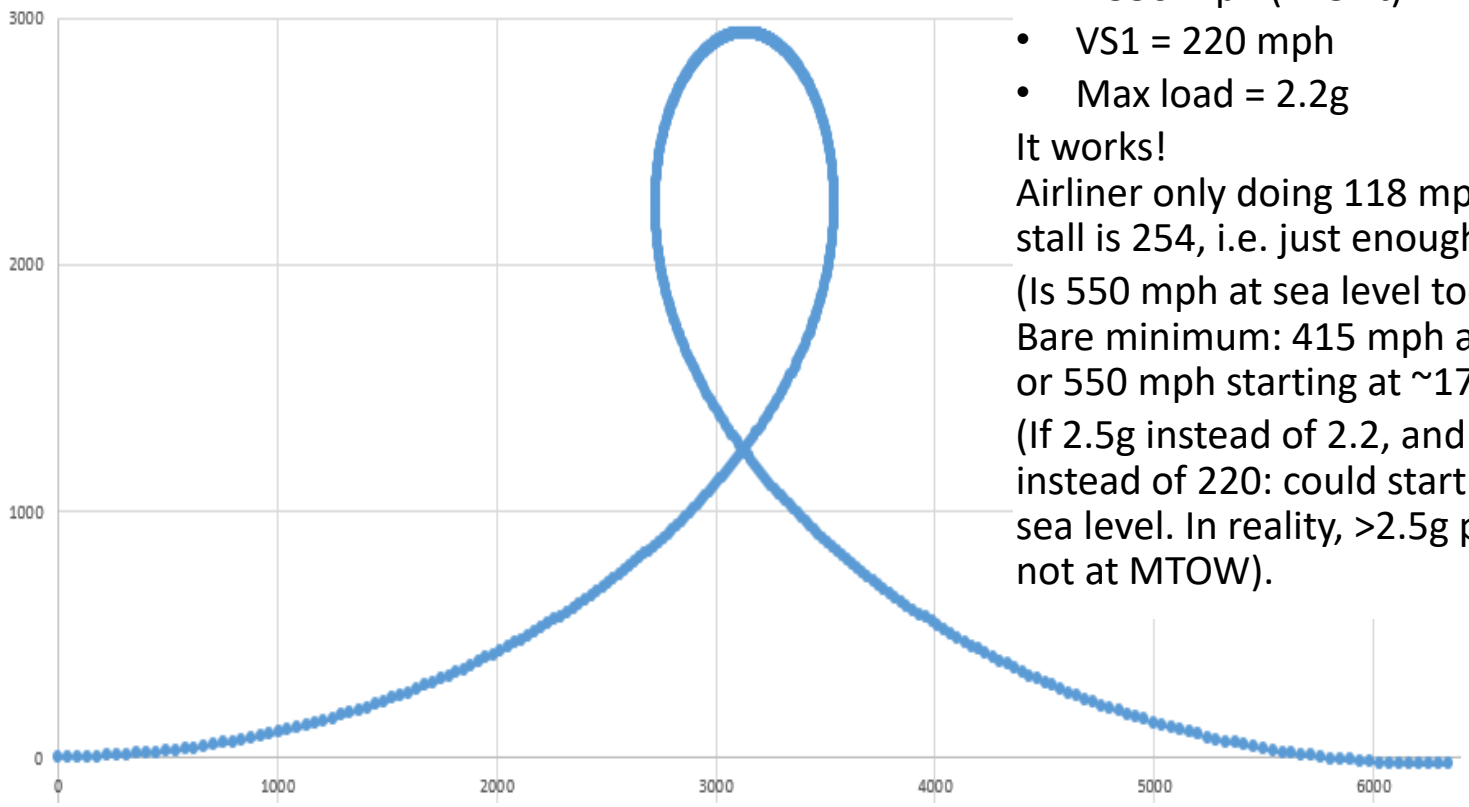
Constants and Conversions	
m/s in an mph	0.44704
radians in a degree	0.017453293
one G in m/s^2	9.81
meters in one foot	0.3048
starting altitude (ft)	0

Segment	Start angle (degrees)		H at start (m)	Start speed (mph)	Gs	Local Radius (m)	Segment Length (m)	Horizontal Component (m)	Vertical Component (m)	1st iteration V at end (m/s)	2nd iteration Local Radius	2nd iteration Segment Length	2nd iteration Horizontal Component	2nd iteration Vertical Component	Density	Stall Speed	Maneuver Speed
0	0		0	550	2.2	5135.283795	44.8137	44.81323633	0.195535872	549.982548	5135.1208	44.81224095	44.81181437	0.195529667	0.0023769	220	326.3127
1	0.5	44.8	53	549.98255	2.2	5134.469142	44.8066	44.80271508	0.586499668	549.930199	5134.1433	44.80371063	44.79987218	0.586462452	0.0023769	220	326.3158
2	1	89.611		549.9302	2.2	5132.514694	44.7895	44.77883935	0.977077751	549.842979	5132.1892	44.78665739	44.77599938	0.977015782	0.00	20.01	326.325
3	1.5	134.388		549.84298	2.2	5129.585031	44.7639	44.74305368	1.367023928	549.720927	5129.26	44.76109548	44.74021849	1.366937306	0.00	0.02	326.3402
4	2	179.128	3.	72094	2.2	5125.682602	44.7299	44.69539186	1.756086729	549.564101	5125.3582	44.72704626	44.69256333	1.755975596	0.	0.03	326.3616
5	2.5	223.82	4.88	6411	2.2	5120.810668	44.6874	44.6358989	2.144015972	549.37257	5120.4871	44.68453818	44.63307887	2.143880516		220.05	326.3891
6	3	268.454	7.0258	58	2.2	5114.973294	44.6364	44.56463086	2.530563129	549.146422	5114.6508	44.63360671	44.56182116	2.530403583	53	220.07	326.4226
7	3.5	313.015	9.5562		2.2	5108.175342	44.5771	44.48165481	2.915481688	548.88576	5107.8541	44.57429424	44.47885725	2.915298326	748	220.1	326.4622
8	4	357.494	12.4715	54	2.2	5100.422461	44.5094	44.38704864	3.298527499	548.590702	5100.1026	44.50665007	44.38426502	3.29832064	23741	220.13	326.5078
9	4.5	401.878	15.7698	548.	2	5091.72108	44.4335	44.28090096	3.679459128	548.261382	5091.4028	44.43073028	44.27813305	3.6792291	0023733	220.17	326.5594
10	5	446.157	19.4491	548.26		5082.078394	44.3494	44.16331091	4.058038198	547.897949	5081.7619	44.34659763	44.16056044	4.05778	0.0023725	220.21	326.617
11	5.5	490.317	23.5068	547.8979		5071.502352	44.2571	44.03438797	4.434029715	547.500568	5071.1878	44.25432146	44.03165665	4.4337	0.0023716	220.25	326.6806
12	6	534.349	27.9406	547.50059													6.75
13	6.5	578.24	32.7475	547.06944													8254
14	7	621.981	37.9245	546.60472													9065
15	7.5	665.559	43.4684	546.10662													9934
16	8	708.964	49.3756	545.57538													0861
17	8.5	752.186	55.6424	545.01123													1845
18	9	795.214	62.265	544.41442													2884
19	9.5	838.037	69.2392	543.78521													398
20	10	880.646	76.5608	543.12389													5131
21	10.5	923.03	84.2251	542.43073													6336
22	11	965.181	92.2276	541.70605													7595
23	11.5	1007.09	100.563	540.95015													8907
24	12	1048.74	109.227	540.16337													0272
25	12.5	1090.13	118.215	539.34604													1689
26	13	1131.25	127.519	538.49851													3156
27	13.5	1172.1	137.136	537.62114													4674
28	14	1212.65	147.06	536.71431													6241
29	14.5	1252.91	157.284	535.77839													7857
30	15	1292.86	167.803	534.81377													9521
31	15.5	1332.5	178.61	533.82086													1231
32	16	1371.83	189.701	532.80006													2988
33	16.5	1410.82	201.067	531.75179													9.479
34	17	1449.49	212.704	530.67649													6636
35	17.5	1487.81	224.604	529.57457													8526
36	18	1525.79	236.761	528.44649													0458
37	18.5	1563.42	249.168	527.29269	2.2	4510.255637	39.3593	37.27048867	12.65162611	526.113577	4510.0388	39.35738864	37.26869712	12.65101796	0.0023207	222.65	330.2432
38	19	1600.69	261.819	526.11363	2.2	4479.712944	39.0927	36.90703243	12.88851271	524.909717	4479.5011	39.09089707	36.90528672	12.88790308	0.0023179	222.79	330.4446
39	19.5	1637.59	274.707	524.90977	2.2	4448.683018	38.822	36.53829746	13.11858925	523.681523	4448.4761	38.82015387	36.53659773	13.11797899	0.002315	222.92	330.65
40	20	1674.13	287.825	523.68158	2.2	4417.186149	38.5471	36.164554	13.34180835	522.429468	4416.9842	38.54533606	36.16290033	13.34119828	0.0023121	223.07	330.8592

Input parameters	
Limit Load (G)	2.2
Stall speed at sea level	220
Speed at bottom (e.g. VNE)	550
Increment size (degrees)	0.5

Density	Stall Speed	Maneuver Speed
0.0023769	220	326.3127
0.0023769	220	326.3158
0.0023768	220.01	326.325
0.0023765	220.02	326.3402
0.0023762	220.03	326.3616
0.0023758	220.05	326.3891
0.0023753	220.07	326.4226

Super Tall Loop



Say…

- Max speed at sea level = 550 mph (478 kt)
- VS1 = 220 mph
- Max load = 2.2g

It works!
Airliner only doing 118 mph at top, stall is 254, i.e. just enough for ~0.2g!

(Is 550 mph at sea level too much? Ok: Bare minimum: 415 mph at sea level, or 550 mph starting at ~17,000 feet).

(If 2.5g instead of 2.2, and 180mph stall instead of 220: could start at 370 mph at sea level. In reality, >2.5g probably ok if not at MTOW).

Slow Rolls

- Level flight,
- knife-edge one way,
- -1g flight,
- knife-edge the other way,
- Back to level flight:

What airplanes can sustain -1g flight?

How about knife-edge?

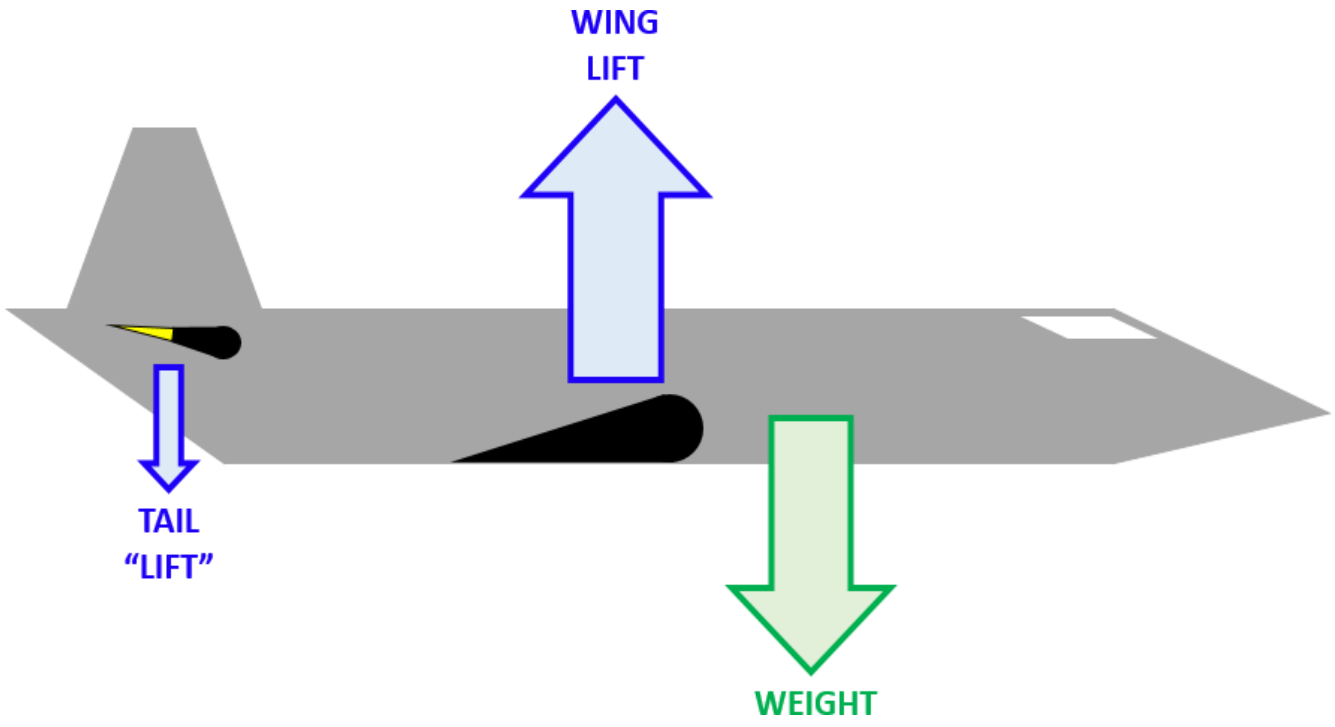
Sustained Inverted Flight (-1g)

… requires three things:

- 1: Structural strength.
- 2: Systems, e.g. inverted oil pump and fuel hose
- 3: Sufficient elevator authority. (This is easier at faster speeds)

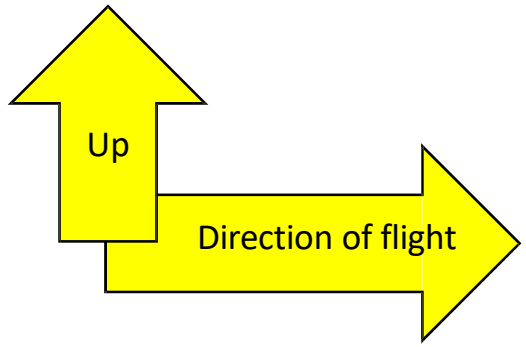
"Elevator Authority"?

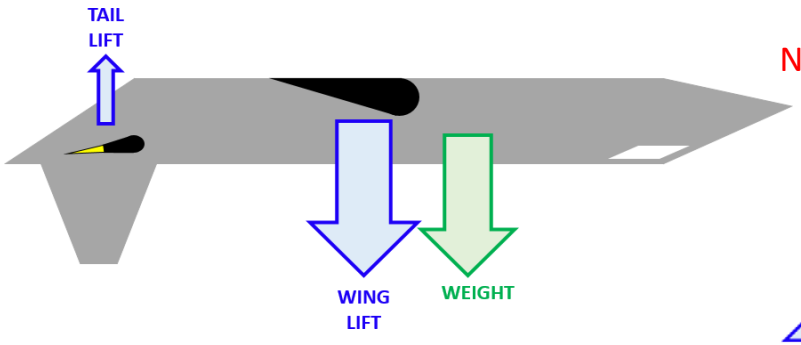
- The first two are obvious. As for the third…
- Recall that the horizontal stabilizer must push down for balance.



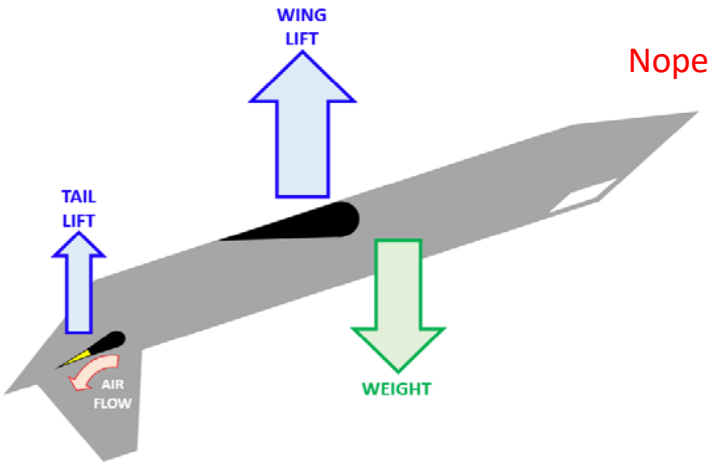
"Elevator Authority"?

- So for sustained inverted flight, we need…

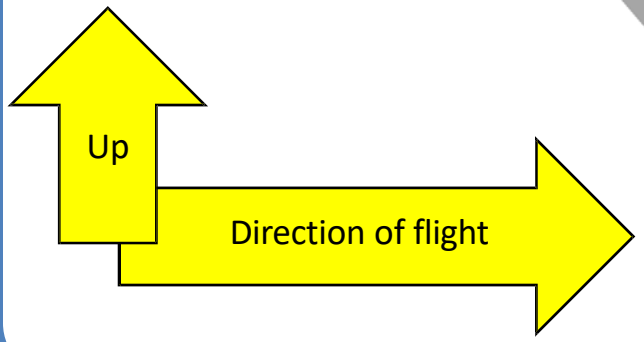


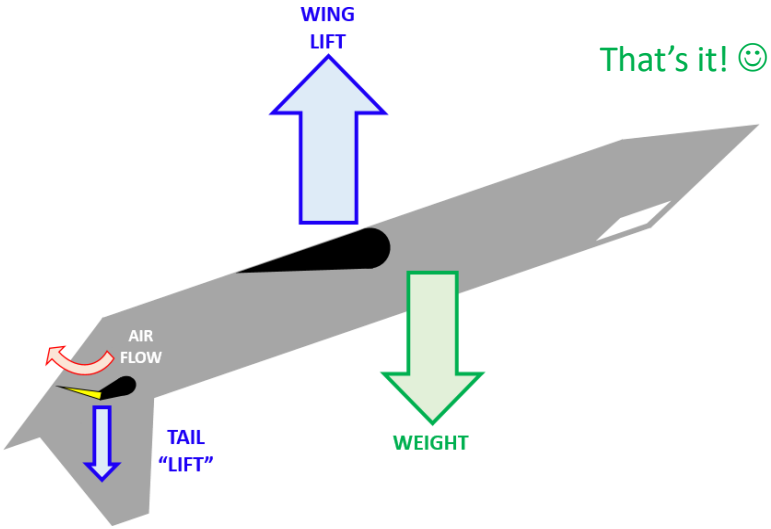


No way! Will fall!



Nope. Nose will come down.



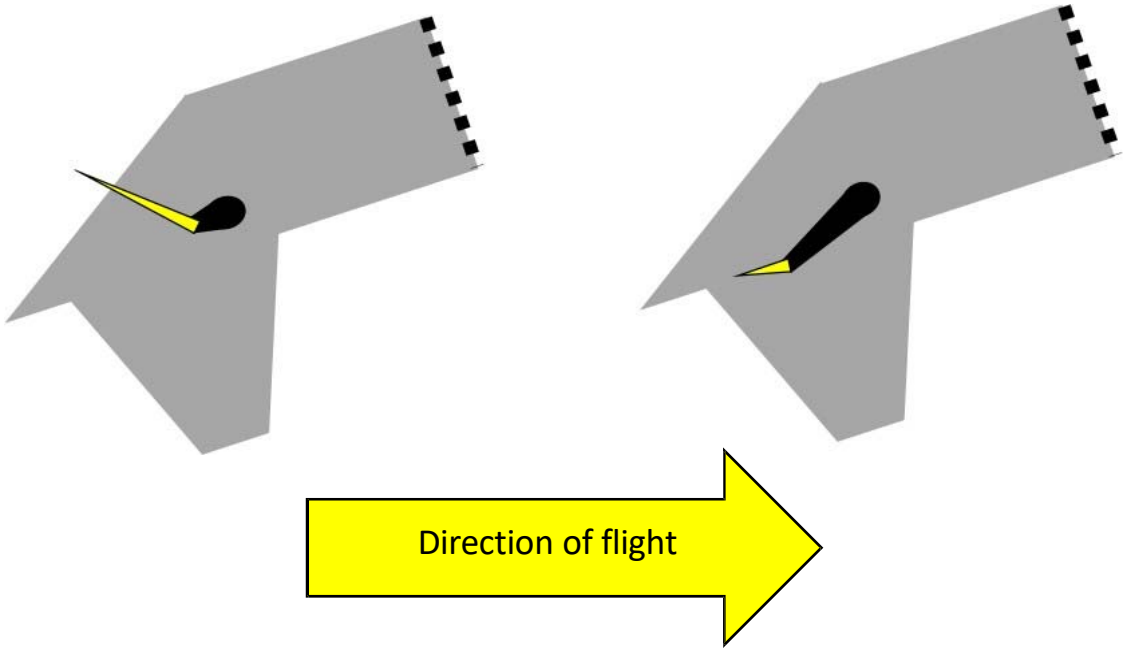


That's it! ☺

"Elevator Authority"?

- So the key questions are: ***Is the elevator big enough***, and does it deflect through ***enough of an angle***, to change the horizontal tail lift from "upwards" to "downwards" when the airplane is at a highly negative angle of attack?
- Note: The faster the speed, the less angle is needed. (Can the airplane do a -1g push-over? If so, then it can sustain -1g flight. And it can, at SOME speed).

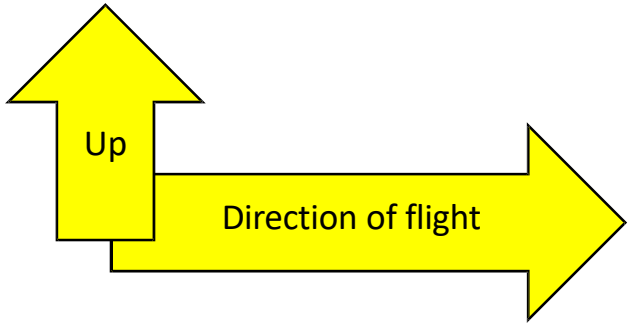
Probably **CAN** sustain -1g flight

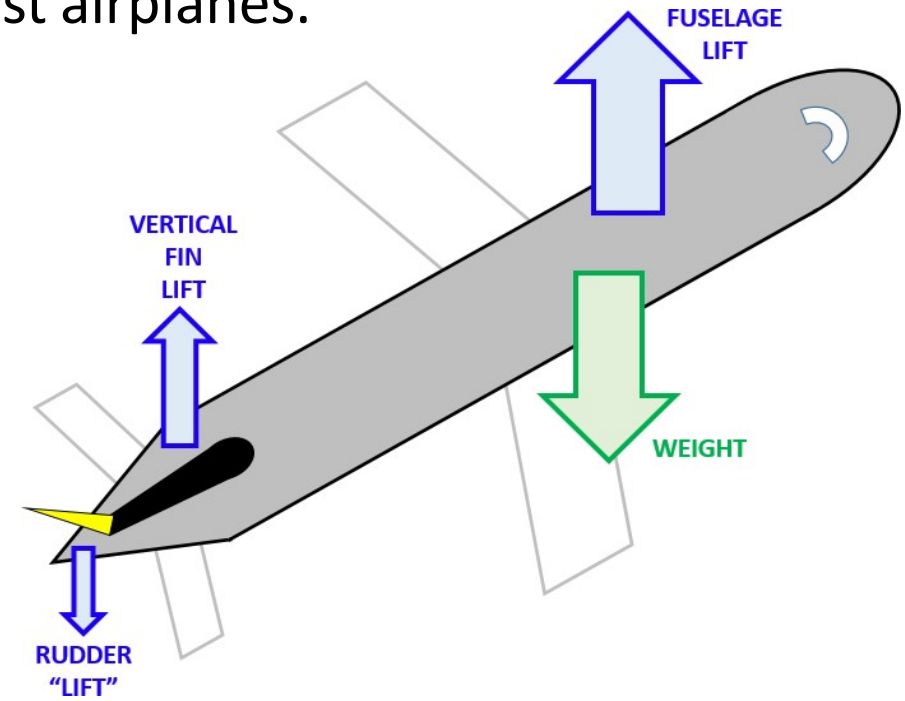


Probably **CANNOT** sustain -1g flight

Knife Edge Flight

- Just as -1g capability depends on elevator authority, knife-edge capability depends on ***rudder authority***.
- Both the CG and the fuselage's center of lift will be somewhere near the front. But the vertical stabilizer will make a lot of lift, wanting to bring the tail up and the nose down. Can the rudder overcome this? Not on most airplanes.
- (i.e. : In airplanes capable of knife-edge flight, a maximum slip – full rudder one way, then bank the other way until the airplane is not turning – will become knife-edge).





Just Scratching the Surface

- We did not discuss some relatively simple maneuvers such as spins, hammerheads, or snap rolls. How could they be modeled?
- Around a loop, Thrust ≠ Drag. By how much? Depends on speed, Gs, relationship between entry speed and max level-flight speed… e.g. an F-22 loop looks very different from a glider loop!
- What about high-torque gyroscopic maneuvers like lomcevaks?
- How are things different in formation aerobatics?
- How about in airplanes that are neutrally stable or unstable, like a Sukhoi-27 performing a Cobra maneuver?
- What possibilities are opened up by thrust-vectoring jets and airplanes with large control surfaces placed in the prop-wash?
- But hopefully tonight's presentation was enough to help you think about the capabilities of a homebuilt to do basic aerobatics.


Questions?